As per Revised Syllabus of

VISVESVARAYA TECHNOLOGICAL UNIVERSITY

ELEMENTS OF CIVIL ENGINEERING AND MECHANICS - *at a Glance*

(One day Revision Book)

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1 Introduction to Civil Engineering

Chapter at a Glance

- Surveying deals with various types of measurements and plotting of maps/plans after applying necessary corrections to the measurements.
- For various types of buildings and other constructions, the materials used are cement, bricks, stones, sand, steel etc.
- Transportation engineering is the discipline that deals with study of present (existing) transportation systems and their improvement for safe, economical and efficient (in less time) conveyance of materials/goods/finished products as well as human beings and animals.
- Hydraulics is an applied science that studies pipe flows and open channel flows, dams and irrigation, hydropower, hydrodynamic machines, fluid control circuitry, etc.
- Water resources and irrigation engineering deals with tapping or storage of water and supplying water either for crop cultivation or for drinking and other domestic/industrial uses.
- Structural engineering is a branch of civil engineering that includes safe and economical design of structures and structural members as well as connections such as rivets, bolts, keys, welds, etc.
- Geotechnical engineering deals with study of soil properties and engineering behaviour of soil under the action of particular loads and moisture content.
- Environmental engineering deals with study of the natural environment/ecosystems, inter-relation between biotic and abiotic factors, safety of people against different types of pollution and treatment-disposal of wastes.

- **Role of Civil Engineer**

Sr. No.	Role	Relevant disciplines	Activities
1)	Surveyor/planner.	Surveying (levelling and contouring also), cartography, building drawing (knowledge of building bye-laws)	Taking measurements, applying necessary corrections to them, preparing maps/plans, selection of site, overall planning of project.
2)	Quantity surveyor/valuer.	Quantity surveying, basics of construction/building drawing.	Determining quantities of materials of construction, rate analysis, study of specifications/contracts, valuation.
3)	Designer/structural consultant.	Structural engineering, soil mechanics earthquake engineering (Design of R.C.C./steel structures, strength of materials, theory of structures etc.)	Finding loads on members, design of foundations and structural members, connections of members, preparing structural drawings.
4)	Contractor/site engineer/site supervisor.	Construction technology/engineering, testing of materials.	Overall quality control, testing of materials for strength, getting construction completed as per the drawings within time and least wastage.
5)	Maintenance contractor/engineer	Construction technology/construction engineering, building design and drawing, sanitary (environmental) engineering, quantity surveying, testing of materials, earthquake engineering.	Visiting the site, proposals and estimate for repair works, (if required, structural design and drawing), completion of work (with precautions to avoid accidents / damages to the people / surrounding buildings) within stipulated time.
6)	Miscellaneous works/role	Transportation engineering, water resources engineering (fluid mechanics, irrigation engineering, hydraulic machines), Infrastructure Development, Environmental engineering.	Disaster management in case of natural calamity such as flood, earthquake. Town planning, traffic design, roads, railways, bridges, tunnel construction/repair. Water and waste water treatment, power generation etc.

- **Types of roads based on function :** Expressways, national highways, state highways, major district roads, other district roads and village roads.
- **Types of roads based on material used for construction :** Earth roads, water bound Macadum roads, bituminous roads and cement concrete roads.
- Usually an artificially constructed way and means for crossing a channel or flowing water course above it is termed as 'bridge'.

- Dam is a barrier or obstacle constructed across a water stream (river) so that water gets accumulated on the upstream side of the barrier.

Important Theory Questions and Answers

Q.1 *Explain briefly the role of civil engineer in the infra-structural development of nation.*

VTU : Feb.-08, Marks 5, Aug.-10, Marks 6, June-12, Marks 6

Ans. : Broadly, infrastructure development involves five stages. viz. planning, designing, costing (estimation), execution and repair/maintenance.

1) Planning

This stage includes following steps and role of civil engineer :

Role as a planner / project consultant / surveyor : **i)** Selection of proper site (plot) **ii)** Preparing site plan **iii)** Taking field measurements by using the knowledge of surveying and levelling, **iv)** Preparing campus plan and building plan are the activities involved in the first stage of construction project.

The plans have to obey the building byelaws so that they are approved/sanctioned by local authority.

2) Designing

After preparing drawings (plans) according to the requirements of owner, role of civil engineer is structural designer or structural consultant. This requires consideration for all types of loads on the structure (viz. dead loads, live loads, seismic loads, etc.) and recommend suitable cross section of structural members such as foundation (footing), beams, columns, etc. and also use proper material to achieve safety and economy. Structural drawings are also prepared. Thus geotechnical and foundation engineering, earthquake engineering and structural engineering are required, so that work is as per Bureau of Indian Standards.

3) Estimating/Costing

This can be the preceding stage or parallel to stage 2 mentioned above. As a valuer or quantity surveyor, civil engineer plays role at this stage. Quantities of construction materials required, rate analysis for them and total amount (cost) of project is worked out with provisions for possible increase in rates (inflation).

This also includes 'phasing' of the project, freezing the specifications for quality of work, preparing tender documents and details of contracts for execution.

4) Execution

As this stage means actual construction, civil engineer has role as a contractor and/or site engineer/supervisor. Earlier stages involve 'proper planning and design' only but unless 'implementation' is properly done, we do not get the expected results and/or advantages. This includes storage and use with least wastage of materials. Civil engineer has to get the work done according to the drawings and specifications. Hence testing of construction materials for their desired/expected properties is obligatory. e.g. quality of bricks, strength of cement and concrete, strength of steel bars used for R.C.C. work, etc. He has to ensure best possible workmanship either as consultant/site engineer or contractor.

Thus civil engineer has to minimise wastage and execute the work in time with systems approach. Critical Path Method (C.P.M.), Bar Chart, Scheduling are useful techniques used in execution of the work in time which automatically saves time, labour and money directly or indirectly.

5) Repair/Maintenance

This is another important stage in which civil engineer plays crucial role for saving material, structural members and/or human lives. Minor repair/maintenance may be for plumbing, painting, damp proofing, etc. (of course this is not the stage required for any new construction.)

There is an expected performance time or guarantee/expiry period for all items/works such as plumbing, wiring, etc. and for the whole structure. Civil engineer as a contractor has to ensure that proper repairing is done without much risk for occupants and surrounding structures. If necessary, the building has to be evacuated and then major repairs can be undertaken. For new construction, old building has to be demolished before or at the 'expiry' time. (e.g. 60 years for building).

Q.2 *Explain different types of roads.*

VTU : Feb.-08, Marks 7; Feb.-09, Aug.-11, June-12, Marks 6

Ans. :

(A) Classification based on significance or function of roads

1) **Expressways :** There are special category of roads constructed for uninterrupted transport. i.e. roads without 'puncture.' Speed limit on such highways are above 60 km/h and are usually constructed in heavy traffic areas. They have advantages like reduction of travel time and less fuel consumption. e.g. Pune-Mumbai expressway.

2) **National Highways (NH) :** National highways are the main highways running through the length and breadth of the country. These highways connect national capital to state capitals, major port towns, border areas, etc. They are the main arteries of transport in the country and are of a strategic importance.

National highways should have at least two, lane traffic and should have high class surface finishing together with adequate structural strength.

e.g. Delhi-Amritsar-Wagha Border is NH1, Agra-Nasik-Mumbai is NH3.

3) **State Highways (SH) :** These are road connecting national highways of a neighbouring states, to district headquarters and major cities in the state. These type of roads are important routes within the state. State highways should be preferably be two lane wide but if it is of one lane only, it must be provided with wide shoulders.

   e.g. Dhule-Aurangabad is SH22. Jalgaon-Dhule is SH6.

4) **Major District Roads (MDR) :** These are the roads connecting important towns, areas of production and market places, connecting with each other or with the main highway of a district. These roads should be metalled and at least one lane wide.

5) **Other District Roads (ODR) :** These are roads connecting a particular town to a town or a village with some other roads. They have some lower specifications than major district roads and state highways. These roads should have metalled surface, proper drainage and should be accessible throughout the year.

6) **Villages Roads (VR) :** These roads are connecting a village to a village or a village to a district or a taluka headquarters. They are meant mainly for village use. These roads can be of earth, but if metalled surface is provided, it can serve for a more useful purpose.

(B) Classification based on materials used in construction

1) **Earth Roads :** These type of roads are constructed using earth. This type of road is the cheapest one and is used in the areas where traffic is less or rare. They are constructed from the locally available soil near the site. The performance of these roads mainly depends on the effective maintenance and drainage.

2) **Water Bound Macadam Roads (WBM) :** Water bound Macadam road is named after a Scottish Engineer John Macadam. Water Bound Macadam is a better quality road than the ordinary earth. A dense and compacted course of a road pavement is composed of stone aggregates bound together by a thin film of cementing medium consisting of a fine material filler. i.e. gravel (small stones) with cementing properties and containing enough moisture to impart binding properties to bind the aggregates together is called as a water bound Macadam. Binding action in WBM construction is obtained by using stone dust as a filler, in presence of water.

3) **Bitumen or Bituminous Roads :** In this type of a construction, bitumen is used as a binding material. Such type of roads are also called as a black top roads. Initially

a WBM or a earth road is constructed and a layer of bitumen is spread on the top, as a wearing layer (top layer). It acts as a binding material and provides a levelled smooth surface for the road.

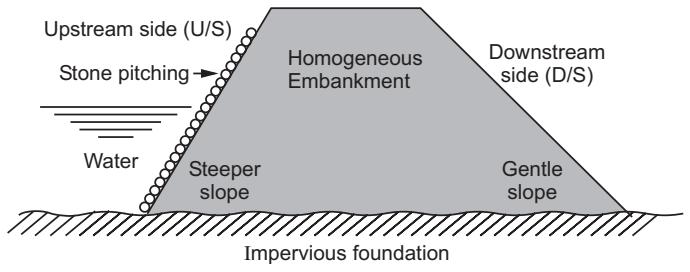
Collectively, above three types fall under **'flexible pavements'** category as their top surface is not rigid (can deform or have flexibility). Repairs are easy but frequent maintenance is required for flexible pavements.

4) **Cement Concrete Roads :** Cement concrete roads are of high standard. They are the costliest of all other types of roads. These roads provide an excellent riding surface and pleasing appearance. They are classified as **'rigid pavements'** as they do not have any flexibility. Hence top surface does not deform. However these pavements can crack and repairing is very difficult. Rigid pavements are virtually maintenance-free.

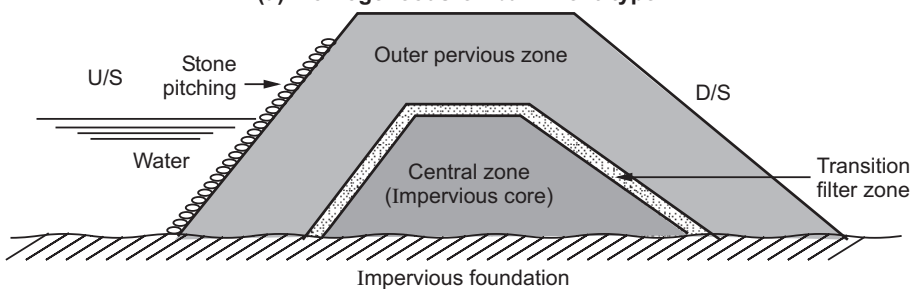
Q.3 *Explain any two types of dams with next figures.*

VTU : Aug.-08, Marks 6; Feb.-10, Marks 8

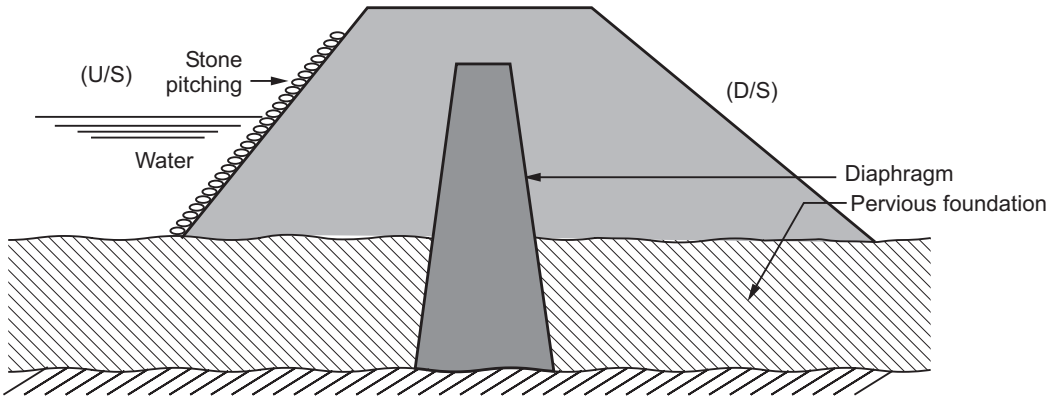
Ans. : **1) Earthen Dams :** Out of the major four types, earthen dam uses good type of soil itself as material of construction, which is well pounded down and compacted. It is useful for the weak foundation conditions and small dams. But there may be problems of seepage and regular careful maintenance of earthen dams. e.g. part of Panshet dam near Pune (Maharashtra) and Nargarjunsagar(Andhra Pradesh), etc. Earthen dams may be further divided into homogeneous embankment type, zoned embankment type and diaphragm embankment type. Refer Fig. 1.1 for these types of dams.



(a) Homogeneous embankment type



(b) Zoned embankment type



(c) Diaphragm type embankment

Fig. 1.1 Earthen dams

2) Rock-fill Dams

These types of dam have characteristics intermediate between earthen and gravity dams. Fig. 1.2 shows typical section of rock-fill dam. An impervious layer slab(usually concrete) has supporting embankment with loose rock-fill on D/S side and stone or dry rubble masonry on U/S side. Self weight of rockfill dam is intermediate between earthen and gravity dams and is of better earthquake, resistance.

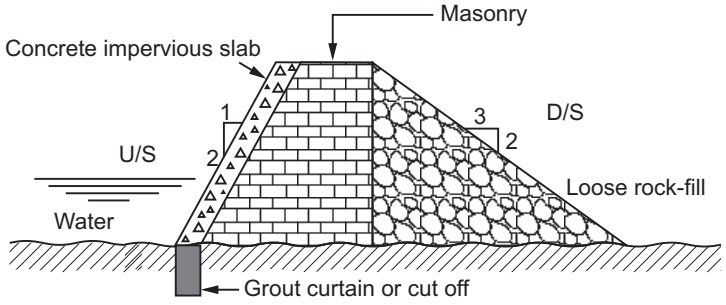


Fig. 1.2 Rock-fill dam

Q.4 *Bring out briefly scope of following specialization of civil engineering :*

i) Structural engineering ii) Transportation engineering.

VTU : Aug.-09, Feb.-10, Marks 8

Ans. : i) Structural Engineering : It is a branch of civil engineering that includes safe and economical design of structures and structural members as well as connections such as rivets, bolts, keys, welds, etc.

For a given loading, suitable cross section of members (beams, columns, etc.) can be determined. Structural engineering further includes engineering mechanics, strength of

materials, theory of structures, design of steel and Reinforced Cement Concrete (R.C.C) structures, etc.

For the given loading, conventional materials as well as alternate/modern materials are used to get minimum possible size of structural members with adequate factor of safety viz. prestressed concrete.

Applications :

1) Design and erection/construction of structural members as well as structures, connectors/fixtures with adequate factory of safety and economy.
2) Design of superstructure and substructure (foundation) of a building or factory shed, etc.
3) Investigation of failure of a member or structure for assigning responsibility and for avoiding repeatation of the mistakes/causes of failure.
4) Design of steel components and structures including water tanks.
5) Design of concrete members, rigid pavements, bridges, etc.

ii) Transportation Engineering : This is the discipline that deals with study of present (existing) transportation systems and their improvement for safe, economical and efficient (in less time) conveyance of materials/goods/finished products as well as human beings and animals. It includes design, construction and management of roads, railways, navigation and air-routes. Allied constructions such as tunnels, bridges, culverts, aqueducts, are also covered in the **sub-disciplines** such as bridge engineering, highway engineering, tunnelling, etc.

Traffic management including traffic signals, number of lanes (and width of roads), parking facilities and curves are also a part of transportation engineering. Applications of transportation engineering are highlighted by today's problems of congestion, road accidents, heavy traffic in urban areas and disaster management (to have effective and efficient transportation of people in case of emergency situations such as floods, war, earthquakes, hurricanes, etc.).

Applications :

1. Design and construction of different types of roads.
2. Traffic management and parking facilities.
3. Design and provision of curves and allied structures such as bridge, culvert, tunnel, ghat-roads.
4. Survey, design and provision of different modes of transportation. viz. airports, railways, roads, ports and harbours, etc.

5. Use modern techniques of management to ensure rapid transportation of people and goods/raw materials/agricultural produce with sufficient convenience, comfort, economy and safety.
6. Avoid heavy traffic through cities/villages by providing bye-pass/diversion roads, expressway.
7. Help the eceonomic growth of regions and country through fast transportation system.
8. Provide durable and strong as well as safe modes of transport and repair/maintenance with least possible delays and inconvenience.

Significance of various modes of transportation depending on number of users has following sequence :

a) Roadways b) Railways c) Airways d) Navigation (water ways)

Important Multiple Choice Questions

Q.1 *Geotechnical engineering involves the study of _____.*

a) water *b) soil* *c) air* *d) all the above*

VTU : Feb.-09, 11, Aug.-11, June-12

(Ans. : b)

Q.2 *By-pass road is constructed _____ .*

a) inside the city *b) over the main road*

c) around the city *d) none of the above* **VTU : Feb.-09**

(Ans. : c)

Q.3 *The part of civil engineering which deals with waste water and solid waste is called _____ .*

a) water supply engineering *b) geotechnical engineering*

c) sanitary engineering *d) structural engineering*

VTU : Feb.-09, June-12

(Ans. : c)

Q.4 *A bascule bridge is a _____ .*

a) floating bridge *b) arch bridge*

c) suspension bridge *d) movable bridge*

VTU : Feb.-09, Feb.-11, June-12

(Ans. : d)

Q.5 *A branch of civil engineering dealing with the technical measures to use and protect the components of environment is _____.*

a) transportation engineering *b) hydraulics engineering*

c) geotechnical engineering *d) environmental engineering*

VTU : Aug.-09
(Ans. : d)

Q.6 *Composite material consisting of cement concrete and steel used in civil engineering structural construction is _____ .*

a) Prestressed Concrete (PSC) *b) Reinforced Cement Concrete (RCC)*

c) Fibre Reinforced Concrete (FRC) *d) Plain Cement Concrete (PCC)*

VTU : Aug.-09
(Ans. : b)

Q.7 *Highways which are superior to national highways and are provided wherever volume of traffic is very high are _____ .*

a) state highways *b) roadways*

c) airways *d) expressways*

VTU : Aug.-09, Aug.-11
(Ans. : d)

Q.8 *A bridge constructed at some angle to river flow is _____ .*

a) skew bridge *b) square bridge* *c) steel bridge* *d) lift bridge*

VTU : Aug.-09, Feb.-11
(Ans. : a)

Q.9 *The part of civil engineering which deals with design of slabs, beams columns footings etc is called _____ .*

a) transportation engineering *b) structural engineering*

c) geotechnical engineering *d) water supply engineering*

VTU : Feb.-10
(Ans. : b)

Q.10 *The science of map making is known as _____ .*

a) estimation *b) surveying*

c) town planning *d) construction technology*

VTU : Feb.-10
(Ans. : b)

Q.11 *The structure which provides passage over the obstacles like valley, river without closing the way underneath is _____.*

a) dam *b) bridge* *c) harbour* *d) airport*

VTU : Feb.-10, Aug.-11
(Ans. : b)

Q.12 *Pick up a structure in which a inspection gallery is formed _____.*

a) gravity dam *b) bridge* *c) harbour* *d) airport*

VTU : Feb.-10, June-12

(Ans. : a)

Q.13 *Effects of pollution and reducing the ill-effects is studied under _____.*

a) surveying *b) geotechnical engineering*

c) hydraulics *d) environment engineering*

VTU : Feb.-10

(Ans. : d)

Q.14 *Land use planning, zoning etc. are a part of _____ .*

a) transportation engineering *b) construction technology*

c) town planning *d) geotechnical engineering*

VTU : Feb.-10

(Ans. : c)

Q.15 *River flood controlling can be done by constructing _____.*

a) bridge *b) tunnel* *c) dam* *d) harbour*

VTU : Feb.-10

(Ans. : c)

Q.16 *Long span bridges are generally made of _____ .*

a) plain cement concrete *b) pre-stressed concrete*

c) timber *d) pozzolona concrete* **VTU : Feb.-10**

(Ans. b)

Q.17 *Identify the mis-matching item _____ .*

a) TMT bars *b) channel section* *c) water pipes* *d) lime*

VTU : Feb.-10

(Ans. : (d) All other are uses of steel)

Q.18 *Which of the following material is not used for dam construction ?*

a) Hollow bricks *b) Stones* *c) Concrete* *d) Timber*

VTU : Feb.-10

(Ans. : a)

Q.19 *_____ is not a natural construction (building) material.*

a) Plywood *b) Stone* *c) Sand* *d) Timber*

VTU : Feb.-10

(Ans. a)

Q.20 *_____ is used for flood control, navigation and hydropower generation.*

a) Multipurpose dam *b) Detention dam*

c) Timber dam *d) Culvert* **VTU : Feb.-10**

(Ans. : a)

Q.21 *Which of the following is not a term related to 'bridge'?*

a) Approach *b) Pier* *c) Impervious core* *d) Abutment*

VTU : Feb.-10

(Ans. : (c) This term is used in earthen dam)

Q.22 *_____ are constructed on both sides of abutments to prevent scouring for protecting the banks of river.*

a) Wing walls *b) Dams* *c) Piers* *d) Trusses*

VTU : Feb.-10

(Ans. : a)

Q.23 *For crossing a water course or valley in all seasons _____ is essential.*

a) Pathways *b) Flooring* *c) Bridge* *d) Curb*

VTU : Feb.-10

(Ans. : c)

Q.24 *Use of _____ is essential for saving time, labour, money, etc. during a work.*

a) construction technology *b) arch bridge*

c) pipe culvert *d) gravity dam* **VTU : Feb.-10**

(Ans. : a)

Q.25 *Studying the properties of soil in assessing safe bearing capacity is called as _____ .*

a) transportation engg. *b) building materials*

c) estimation *d) geotechnical engineering* **VTU : Aug.-10**

(Ans. : d)

Q.26 *Building tanks and dams and carrying stored water to field is known as _____ .*

a) structural engineering *b) environmental engineering*

c) water resources and irrigation engineering *d) construction technology*

VTU : Aug.-10, 11

(Ans. : c)

Q.27 *Kerbs are the components of _____ .*

a) roads *b) bridges* *c) building* *d) dam*

VTU : Aug.-10

(Ans. : a)

Q.28 *Reinforced Cement Concrete (RCC) comes under _____ .*

a) architecture and town planning *b) hydraulics*

c) surveying *d) structural engineering* **VTU : Aug.-10**

(Ans. : d)

Q.29 *Shoulders are the components of _____ .*

a) roads *b) bridges* *c) buildings* *d) dams*

VTU : Feb.-11

(Ans. : a)

Q.30 *Abuttment is a part of _____ .* **VTU : Dec.-11**

a) road *b) bridge* *c) dam* *d) building*

(Ans. : b)

Q.31 *Which of the following is not an irrigation infrastructure ?* **VTU : Dec.-11**

a) Dam *b) Canal* *c) Jackwell* *d) Road*

(Ans. : d)

Q.32 *Surveying mainly deals with _____ .* **VTU : Dec.-11**

a) communication *b) environment*

c) material *d) measurement*

(Ans. : d)

Q.33 *Geotechnical engineering mainly deals with _____ .* **VTU : Dec.-11**

a) space *b) air* *c) earth* *d) water*

(Ans. : c)

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Notes

2 Introduction to Engineering Mechanics

Chapter at a Glance

- **Force :** It is a physical quantity that changes or tries to change the state of rest or of uniform motion of an object.
- **Particle :** It is an object that has mass but no dimensions.
- **Rigid body :** It can be considered as a combination of large number of particles occupying fixed positions with respect to each other.
- **Continuum :** Bodies are assumed to have continuous distribution of matter even though they are made up of particles like atoms and molecules which is known as continuum.
- **Point force :** If the area of contact between the two objects is small and can be neglected, the force can be considered as a point force.
- **Six fundamental principles of mechanics :**

Newton's first law : Every body tries to be in its state of rest or of uniform motion along a straight line unless it is acted upon by an external unbalanced force.

Newton's second law : The rate of change of momentum of a body is directly proportional to the force acting on it and takes place in the direction of the force.

Newton's third law : Every action has equal and opposite reaction.

Newton's law of gravitation : Every body in the universe attracts every other body with a force whose magnitude is directly proportional to the product of the two masses and inversely proportional to square of the distance between them.

Parallelogram law : If two vectors are represented in magnitude and direction by two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn from the common point.

Law of transmissibility of force : Force can be transmitted from one point to another on its line of action without causing any change in the state of its motion.

Important Theory Questions and Answers

Q.1 *State Newton's three laws of motion.* **VTU : Feb.-07, 09, Dec.-11, Marks 6**

Ans. : Newton's first law : Every body tries to be in its state of rest or of uniform motion along a straight line unless it is acted upon by an external unbalanced force.

Newton's second law : The rate of change of momentum of a body is directly proportional to the force acting on it and takes place in the direction of the force.

Newton's third law : Every action has equal and opposite reaction.

Q.2 *Define the following : i) Particle ii) Rigid body iii) Continuum iv) Force.*

VTU : Aug.-08, Marks 6

Ans. :

i) **Particle :** It is an object that has mass but no dimensions.

ii) **Rigid body :** It can be considered as a combination of large number of particles occupying fixed positions with respect to each other.

iii) **Continuum :** Bodies are assumed to have continuous distribution of matter even though they are made up of particles like atoms and molecules which is known as continuum.

iv) **Force :** It is a physical quantity that changes or tries to change the state of rest or of uniform motion of an object.

Important Multiple Choice Questions

Q.1 *An object regarded as only mass but no size in mechanics is ————.*

a) point force b) rigid body c) deformable body d) particle

VTU : Aug.-09, Feb.-13

(Ans. : d)

Q.2 *Which of the following is the basic concept of mechanics ?*

a) Charge b) Power c) Force d) Energy

VTU : Aug.-11

(Ans. : c)

Q.3 *Which of the following statements is correct for a particle ?*

a) It has both mass and dimensions. b) It has mass but no dimensions.

c) It has dimensions but no mass. d) It does not have either mass or dimensions.

(Ans. : b)

Q.4 *Which of the following statements is incorrect for a force ?*

a) It can produce acceleration. b) It can produce deformation of object.

c) It can change the direction of motion of the object. d) None of the above. **(Ans. : d)**

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3 Force, Moment of Force and Couple

Chapter at a Glance

- Force is the physical quantity that changes or tends to change the state of rest or of uniform motion of an object.
- Every force is characterized by its magnitude, direction and point of application. The direction and point of application together define its line of action.
- The motion of object remains unchanged even if the force is transmitted from one point to another on its line of action i.e., the force can be considered as a sliding vector. This statement is called the **law of transmissibility of force.**
- Coplanar force system consists of forces which have lines of action in the same plane.
- Non-coplanar force system consists of forces whose lines of action do not lie in one plane.
- Concurrent force system consists of forces whose lines of action intersect at a single point. Concurrent force systems can be coplanar or non-coplanar.
- Non-concurrent force system consists of forces whose lines of action do not intersect at a single point.
- Collinear force system consists of forces which act along the same straight line.
- Non-collinear force system consists of forces which do not act along the same line.
- Parallel force system consists of forces which have parallel lines of action.
- When two or more forces act on an object, every force produces its own effect on the object independent of the remaining forces. This is known as principle of physical independence of forces.
- Superposition principle states that if more than one force act on an object, then the combined effect due to all the forces is the vector addition of all the individual effects.

- Resolution is a process in which a force is replaced by two or more forces known as components which together produce the same effect as that produced by the single force.
- Composition is the process in which two or more forces are combined together and replaced by a single force (known as resultant force) which produces the same effect as that produced by all the forces together.
- **Parallelogram law :** It two forces are represented in magnitude and direction by two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn from their common point. If $\vec{P}$ and $\vec{Q}$ are two forces and α is the angle between them then the magnitude of their resultant is given by,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

and the angle 'θ' of $\vec{R}$ with $\vec{P}$ is given by,

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

- **Triangle law :** If two forces are represented in magnitude and direction by two sides of a triangle taken in order, their resultant is represented in magnitude and direction by the third side of the triangle drawn from starting point of first force to end point of second force.
- **Moment of force about a point :** The magnitude of moment about a point is product of the magnitude of force and perpendicular distance of the force from the point. The moment can be either clockwise or anticlockwise.
- Couple consists of two equal, opposite and noncollinear forces.
- Moment of couple is product of any one force and the perpendicular distance between them.

Important Theory Questions

Q.1 *Define force and state its characteristics.* **VTU : Feb.-09, June-13, Marks 5**

Ans. : Force : Force is the physical quantity that changes or tends to change the state of rest or of uniform motion of an object.

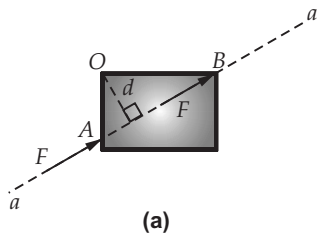
Characteristics of Force : Every force is characterized by its magnitude, direction and point of application. The direction and point of application together define its line of action.

Q.2 *State and explain principle of transmissibility of forces.*

VTU : Feb.-07, 08, 11, Aug.- 08, 12, Marks 4

Ans. : Statement : The motion of object remains unchanged even if the force is transmitted from one point to another on its line of action i.e., the force can be considered as a sliding vector. This statement is called the **law of transmissibility of force.**

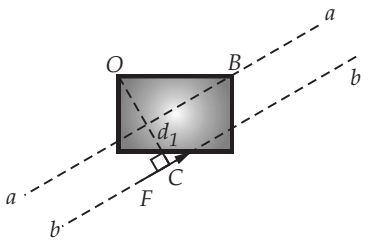
Explanation : If a force F acting at point A is shifted to another point B on its line of action, the perpendicular distance from any point O will remain the same as shown in Fig. 3.1 (a). Therefore the moment of force remains same in magnitude and sense of rotation even if the force is transmitted to any point on line '*aa*'. The linear acceleration is also same for such a shift in the point of application of force.



(a)

Fig. 3.1

If the force is shifted to some point '*C*' which is not on the line '*aa*' as shown in Fig. 3.1(b), then the perpendicular distance from point '*O*' changes. This will change the moment of force about '*O*'. As a result, the angular acceleration will change even though the linear acceleration will remain same.



(b)

Fig. 3.1

Q.3 *State and prove parallelogram law of forces.*

VTU : Feb - 07, Aug.- 08, Marks 5

Ans. : Statement : It two forces are represented in magnitude and direction by two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram drawn from their common point. If $\vec{P}$ and $\vec{Q}$ are two forces and α is the angle between them then the magnitude of their resultant is given by,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

and the angle 'θ' of $\vec{R}$ with $\vec{P}$ is given by,

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

Proof : The two given forces $\vec{P}$ and $\vec{Q}$ shown in Fig. 3.2 (a) are drawn from a common point A as shown in Fig. 3.2 (b). The lengths of AD and AB are proportional to magnitudes of $\vec{P}$ and $\vec{Q}$ respectively. The parallelogram is completed by drawing lines

parallel to AB and AD. The diagonal AC represents the resultant $\vec{R}$. The line AD is extended and a perpendicular is drawn from C on this line.

As $ABCD$ is a parallelogram,

$AB = CD = Q$ (the magnitude of $\vec{Q}$)

$AD = P$ (the magnitude of $\vec{P}$)

$AC = R$ (the magnitude of $\vec{R}$)

$$\angle BAD = \angle CDE$$

$$= \alpha \text{ (the angle between } \vec{P} \text{ and } \vec{Q})$$

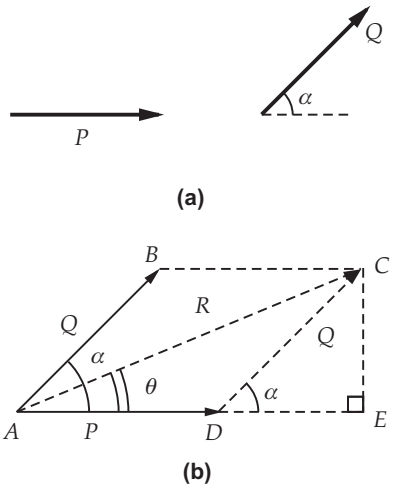


Fig. 3.2

In right angled ΔCDE,

$$DE = CD \cos\alpha = Q \cos\alpha$$

and $$CE = CD \sin\alpha = Q \sin\alpha$$

In right angled ΔACE,

$$AC^2 = AE^2 + CE^2$$

But $$AE = AD + DE = P + Q\cos\alpha,$$

$CE = Q \sin\alpha$ and $AC = R$

$$\therefore \quad R^2 = (P + Q\cos\alpha)^2 + (Q\sin\alpha)^2$$

$$= P^2 + 2PQ\cos\alpha + Q^2\cos^2\alpha + Q^2\sin^2\alpha$$

$$= P^2 + 2PQ\cos\alpha + Q^2(\cos^2\alpha + \sin^2\alpha)$$

$$R^2 = P^2 + 2PQ\cos\alpha + Q^2$$

$$\therefore \quad \boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}}$$

In ΔACE, $\tan\theta = \dfrac{CE}{AE}$

$$\therefore \quad \boxed{\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}}$$

Q.4 *Define a couple. Mention its characteristics.* **VTU : Aug.-11, Marks 4**

Ans. : Definition : Couple consists of two equal, opposite and noncollinear forces.

Characteristics : 1) As the two forces are equal in magnitude but opposite in direction, their resultant force is zero but a moment is associated with couple.

2) A couple acting on a rigid body can produce only rotational motion. It cannot produce translational motion.

3) Moment of couple is same about any point in the plane.

4) The effect of a couple can be nullified only by another couple of equal magnitude but opposite sense of rotation.

Q.5 *State and explain triangle law of forces to find resultant of two forces.*

Ans. : Statement : If two forces are represented in magnitude and direction by two sides of a triangle taken in order, their resultant is represented in magnitude and direction by the third side of the triangle drawn from starting point of first force to end point of second force.

Explanation : To find resultant of two given forces $\vec{A}$ and $\vec{B}$ as shown in Fig. 3.3 (a), $\vec{B}$ is drawn from the end point of $\vec{A}$ and then the triangle is completed by drawing the third side from the starting point of $\vec{A}$ to end point of $\vec{B}$. This third side represents the resultant $\vec{R}$ in magnitude and direction as shown in Fig. 3.3 (b). Triangle law can be used for graphical solution as well as analytical solution.

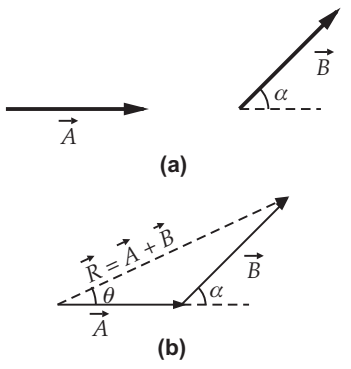


Fig. 3.3

For analytical solutions sine rule or cosine rule can be used after completing the triangle.

Q.6 *Explain resolution and composition of forces cartesian coordinate system.*

Ans. : Resolution : Forces can be resolved into two mutually perpendicular directions (not necessarily horizontal and vertical). The two perpendicular directions can be assumed to be the X and Y axes.

To resolve any force of magnitude F draw lines parallel to the required X and Y-directions from base of the force. The two components will be adjacent to the force.

Find angle say 'θ', of force with either X or Y-axis. The magnitude of component adjacent to that angle θ will be $F\cos\theta$ and the other perpendicular component will be $F\sin\theta$.

The sign of these components will depend on whether they are along positive or negative co-ordinate axes.

If the force is either along X or Y-direction, it will have only one component and the other will be zero. For example, if a force is along X-direction, its Y-component will be zero.

Composition : If F_x and F_y are known, the magnitude and direction of $\vec{F}$ can be obtained using

$$F = \sqrt{F_x^2 + F_y^2}$$

and $$\theta = \tan^{-1}\left(\frac{|F_y|}{|F_x|}\right) \text{ with X-axis}$$

The signs of F_x and F_y are used to determine the quadrant for F.

Method to Solve Examples on Resolution of Forces in Cartesian System

- To resolve any force of magnitude F draw lines parallel to the required X and Y-directions from base of the force. The two components will be adjacent to the force.
- Find angle say 'θ', of force with either X or Y-axis. The magnitude of component adjacent to that angle θ will be $F\cos\theta$ and the other perpendicular component will be $F\sin\theta$.
- The sign of these components will depend on whether they are along positive or negative co-ordinate axes.
- If the force is either along X or Y-direction, it will have only one component and the other will be zero. For example, if a force is along X-direction, its Y-component will be zero.

Important Solved Examples on Resolution of Forces in Cartesian System

Example 3.1 *A force of 200 N is acting on a block as shown in Fig. 3.4 Find the component of force along the horizontal and vertical axis.* **VTU : Feb.-07, 13**

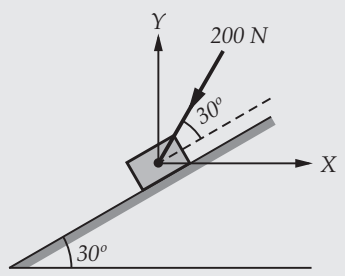


Fig. 3.4

Solution : The angle made by the given force with horizontal is 60°.

$\therefore$ $F_x = -200 \cos 60$

$\therefore$ $\boxed{F_x = -100 \text{ N}}$

$F_y = -200 \sin 60$

$\therefore$ $F_y = 173.2\ N$

Example 3.2 *A block of weight 200 N is kept on the inclined plane and is fixed to the plane. Find the component of weight in the direction along the plane and perpendicular to the plane as indicated (Refer Fig. 3.5).*

VTU : Feb.-12, Marks 4

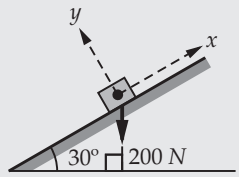


Fig. 3.5

Solution : The components of weight are as shown in Fig. 3.5. (a).

$F_x = -200 \sin 30$

$\therefore$ $F_x = -100\ N$

$F_y = -200 \cos 30$

$\therefore$ $F_y = -173.1\ N$

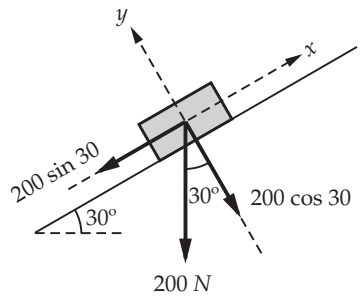


Fig. 3.5. (a)

Method to Find Moment of Force About a Point

- If the perpendicular distance d of the force of magnitude F is known from the given point, use

  $M = F \times d$ to find the magnitude of moment.

To find the sense of rotation, see how the perpendicular rotates about the point when the force is applied.

- If the perpendicular distance is not known for the force, but the horizontal and vertical distances are known for the point of application of force, resolve the force into x and y components and find their combined moment.
- For maximum moment with given force or to produce given moment with minimum force,
  1) force has to be applied at maximum distance from the point about which moment is taken and
  2) force has to be applied perpendicular to the line joining the point about which moment is taken and the point of application of force.

Important Solved Examples on Moment

Example 3.3 *Determine the angle α for which the moment of the 500 N force shown in Fig. 3.6 is maximum about O. Also find the maximum moment.*

VTU : Feb.-11, Marks 10

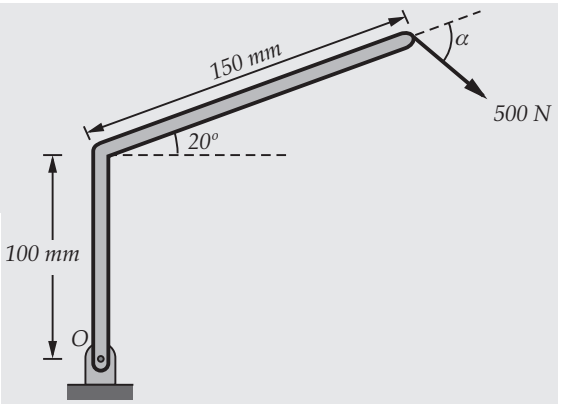


Fig. 3.6

Solution : Moment of the 500 N force will be maximum about *O* when it is applied perpendicular to line *OA* as shown in Fig. 3.6 (a). Length *OA* can be obtained using cosine rule

$$\cos 110 = \frac{150^2 + 100^2 - OA^2}{2(150)(100)}$$

$$OA = 206.79 \text{ mm}$$

$$\therefore \quad OA = 0.20679 \text{ m}$$

Using sine rule,

$$\frac{100}{\sin\beta} = \frac{206.79}{\sin 110}$$

$$\therefore \quad \beta = 27.03^o$$

From Fig. 3.6 (a), $\alpha + \beta = 90$

$$\alpha = 90 - 27.03$$

$$\therefore \quad \boxed{\alpha = 62.97^o}$$

$$M_{max} = 500 \times 0.20679$$

$$\therefore \quad \boxed{M_{max} = 103.395 \text{ Nm}}$$

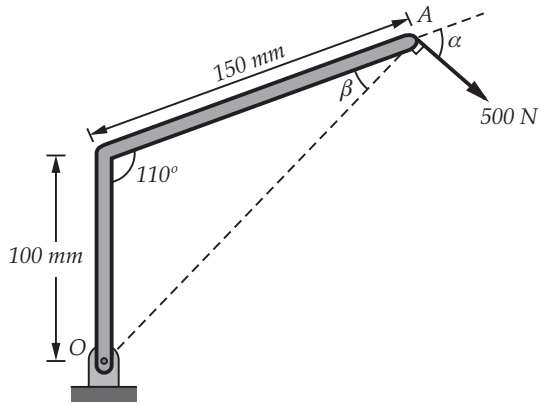


Fig. 3.6 (a)

Example 3.4 *A 100 N verticle force is applied to the end of a lever which is attached to a shaft as shown in Fig. 3.7, determine*

i) The moment of force about O.

ii) The horizontal force applied at 'A' which creates same moment about 'O'.

iii) The smallest force applied at 'A' which creates same moment about 'O'.

VTU : Aug.-11, Marks 6

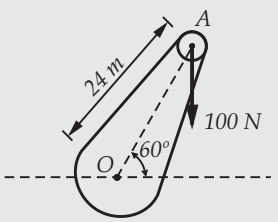


Fig. 3.7

Solution : i) M_O = (100) (2.4 cos 60) ↺

$\therefore$ M_O = 120 Nm ↺

ii) Let F be a horizontal force at A

$\therefore$ 120 = $F \times$ 2.4 sin 60

$\therefore$ F = 57.735 N →

iii) The smallest force 'P' at A is perpendicular to OA. Hence it makes angle 30° with horizontal. The perpendicular distance will be 2.4 m.

$\therefore$ 120 = $P \times$ 2.4

$\therefore$ P = 50 N, 30° ↙

Important Multiple Choice Questions

Q.1 *Moment of a force can be defined as the product of force and _____ distance from the line of action of force to the moment centre.*

a) least *b) maximum* *c) any* *d) none of the above*

VTU : Feb.-09, June-13

(Ans. : a)

Q.2 *Effect of force on a body depends on _____.*

a) direction *b) magnitude* *c) position* *d) all the above*

VTU : Feb.-09, 10, Aug.-11, 12, June-13

(Ans. : d)

Q.3 *The forces which meet at one point and have their lines of action in different planes are called ____.*

a) coplanar concurrent forces *b) noncoplanar concurrent forces*

c) non coplanar non-concurrent forces *d) none of the above.*

VTU : Feb.-09, June-13

(Ans. : b)

Q.4 *Couple means two forces acting parallel, _____.*

a) equal in magnitude and in the same direction

b) not equal in magnitude but in the same direction

c) equal in magnitude but opposite in direction.

d) none of the above

VTU : Feb.-09, Aug.-12, June-13

(Ans. : c)

Q.5 *Forces whose line of action lie along the same line are _____.*

a) coplanar parallel forces *b) collinear forces*

c) coplanar concurrent forces *d) coplanar non-concurrent forces.*

VTU : Aug.-09

(Ans. : b)

Q.6 *Moment of a force about a moment centre is the measure of its _____.*

a) translatory effect b) rotational effect c) both a and b d) none of the above

VTU : Aug.-09, 12, Jan.-13

(Ans. : b)

Q.7 *The translatory effect of a couple on the rigid body is _____.*

a) positive *b) negative* *c) zero* *d) none of the above.*

VTU : Aug.-09, Jan.-13

(Ans. : c)

Q.8 *Principle of transmissibility of forces states that when a force acts upon a body, its effect is ____.*

a) minimum if it acts at the C.G. of the body

b) maximum if it acts at the C.G. of the body

c) same at every point on its line of action

d) different at different points on its line of action. **VTU : Feb.-10, Aug.-10**

(Ans. : c)

Q.9 *Two parallel forces equal in magnitude and opposite in direction and separated by a finite distance are said to form ____ .*

a) moment *b) couple* *c) resultant* *d) equilibriant*

VTU : Feb.-10

(Ans. : b)

Q.10 *The forces which pass through a single point and lie in the same plane are _____ .*

a) collinear forces *b) coplanar non-concurrent forces*

c) coplanar concurrent forces *d) none of these* **VTU : Feb.-10**

(Ans. : c)

Q.11 *Every force acting on an object produces its own effect independent of the other forces. This is known as _____ .*

a) principle of transmissibility of force

b) superposition principle

c) principle of physical independence of forces

d) none of the above

(Ans. : c)

Q.12 *The components of a force vector are _____ .*

a) scalars *b) vectors* *c) zero* *d) none of the above*

(Ans. : b)

Q.13 *Couple is a _____ .*

a) free vector *b) sliding vector* *c) fixed vector* *d) unit vector*

(Ans. : a)

Q.14 *Force can be resolved into _____ .*

a) only one component *b) only two components*

c) only three components *d) any number of components*

(Ans. : d)

Q.15 *State which of the following statements is true ?*
Two forces can produce the same moment about a point _____ .

a) only when the two forces are equal

b) only when the moment arms of the forces are equal

c) when the two forces are concurrent

d) when the product of the force and the moment arm are equal

(Ans. : d)

Q.16 *The magnitude of the moment is _____ when a force is applied perpendicular to a lever.*

a) maximum *b) minimum* *c) zero* *d) negative* **VTU : Aug.-12**

(Ans. : a)

Q.17 *The magnitude of the moment is maximum when a force is applied _____ the lever.*

a) parallel to *b) inclined to* *c) perpendicular to* *d) at the hinge of*

(Ans. : c)

Q.18 *A 500 N force makes an angle of 30° with the y-axis in second quadrant. Its x-component is _____.*

a) – 250 N *b) 250 N* *c) 250 $\sqrt{3}$ N* *d) – 250 $\sqrt{3}$ N*

(Ans. : a)

Q.19 *A force of magnitude 10 N makes an angle of 210° with the positive x-axis. Its x-component will be _____.*

a) – 5 N *b) 5 N* *c) – 5 $\sqrt{3}$ N* *d) 5 $\sqrt{3}$ N*

(Ans. : c)

Q.20 *The x and y components of a force are 10 N and – 10 N respectively. The angle made by the force with positive x-axis is _____.*

a) 45° *b) 135°* *c) 225°* *d) 315°*

(Ans. : d)

Q.21 *The x-component of a 10 N force is 5 N. The angle made by the force with y-axis is _____.*

a) 0^o *b) 30^o* *c) 45^o* *d) 60^o*

(Ans. : b)

Q.22 *A force of magnitude 100 N is directed from point $A(1, 3)$ to $B(4, 7)$. The x and y components of force are _____.*

a) 60 N, 80 N *b) 80 N, 60 N* *c) – 80 N, 60 N* *d) 80 N, – 60 N*

(Ans. : a)

Q.23 *A force of magnitude 50 N is directed from $A(1, 1)$ to $B(-3, -2)$. The x and y components of force are _____.*

a) 40 N, 30 N *b) – 40 N, – 30 N* *c) – 40 N, 30 N* *d) 40 N, – 30 N*

(Ans. : b)

Q.24 *A block weighing 10 kN rests on an inclined plane of inclination 30^o with the horizontal. The magnitude of the component of weight parallel to inclined plane is _____.*

a) 5 kN *b) $5\sqrt{3}$ kN* *c) 10 kN* *d) $10\sqrt{3}$ kN*

(Ans. : (a) The component parallel to plane is W sin θ)

Q.25 *A force of magnitude 26 N is acting along a line of slope $\frac{12}{5}$. The magnitude of the vertical component of the force is _____.*

a) 24 N *b) 10 N* *c) 20 N* *d) 65 N*

(Ans. : a)

Q.26 *The horizontal component of force in Q.25 is, _____.*

a) 24 N *b) 10 N* *c) 20 N* *d) 65 N*

(Ans. : b)

Q.27 *The component of a force $\vec{F} = (10\ \hat{i} + 20\ \hat{j})$ N along a vector $\vec{a} = 3\ \hat{i} + 4\ \hat{j}$ is _____.*

a) 30 N *b) 20 N* *c) 25 N* *d) 22 N*

(Ans. : d The component is $\vec{F} \cdot \frac{\vec{a}}{|\vec{a}|}$)

Q.28 *A force of magnitude 100 N makes an angle of 320^o with the positive x-axis. The x and y-components of the force are _____.*

a) 100 cos 320 N, 100 sin 320 N *b) 100 cos 320 N, – 100 sin 320 N*

c) – 100 cos 320 N, 100 sin 320 N *d) – 100 cos 320 N, – 100 sin 320 N*

(Ans. : a)

Q.29 *A force of magnitude 50 N makes an angle of 150^o with the positive x-axis. The x and y components of force are _____.*

a) $25\sqrt{3}$ N, 25 N *b) $-25\sqrt{3}$ N, 25 N*

c) 25 N, $25\sqrt{3}$ N *d) – 25 N, $25\sqrt{3}$ N*

(Ans. : b)

Q.30 *If two forces $\vec{F}_1$ and $\vec{F}_2$ are perpendicular to each other,* _____.

a) $\vec{F}_1 + \vec{F}_2 = 0$ *b)* $\vec{F}_1 - \vec{F}_2 = 0$ *c)* $\vec{F}_1 \times \vec{F}_2 = 0$ *d)* $\vec{F}_1 \bullet \vec{F}_2 = 0$

(Ans. : d)

Q.31 *If two forces $\vec{F}_1$ and $\vec{F}_2$ are parallel to each other,* _____.

a) $\vec{F}_1 + \vec{F}_2 = 0$ *b)* $\vec{F}_1 - \vec{F}_2 = 0$ *c)* $\vec{F}_1 \times \vec{F}_2 = 0$ *d)* $\vec{F}_1 \bullet \vec{F}_2 = 0$

(Ans. : c)

Q.32 *The component of force is always* _____.

a) less than its magnitude *b) greater than its magnitude*

c) equal to its magnitude *d) none of the above*

(Ans. : d)

Q.33 *The moment of a 100 N force acting at point A (4, 4) is zero about the point B (1, 1). The angle made by the 100 N force with x-axis is* _____.

a) 0^o *b)* 30^o *c)* 45^o *d)* 60^o

(Ans. : (c) The force must be directed along AB.)

Q.34 *The moment of a 50 N force acting at point A(1, – 4) about a point* B $(-5, -4)$ *is zero. The angle made by the 100 N force with x-axis is* _____.

a) 0^o *b)* 30^o *c)* 45^o *d)* 60^o

(Ans. : a)

Q.35 *Angle made by the force $\vec{F} = 5\,\hat{i} + 6\,\hat{j}$ with y-axis is* _____.

a) $\tan^{-1}\left(\frac{6}{5}\right)$ *b)* $\tan^{-1}\left(\frac{5}{6}\right)$ *c)* $\sin^{-1}\left(\frac{6}{5}\right)$ *d)* $\cos^{-1}\left(\frac{6}{5}\right)$

(Ans. : b)

Q.36 *Angle made by the force* $\vec{F} = 3\,\hat{i} + 4\,\hat{j}$ *with y-axis is* _____.

a) $\tan^{-1}\left(\frac{4}{3}\right)$ *b)* $\sin^{-1}\left(\frac{4}{5}\right)$ *c)* $\sin^{-1}\left(\frac{3}{5}\right)$ *d)* $\cos^{-1}\left(\frac{3}{5}\right)$

(Ans. : c)

Q.37 *The physical quantity that produces translational motion is* _____ .

a) force *b) energy* *c) momentum* *d) none of the above*

(Ans. : a)

Q.38 *The physical quantity that produces rotational motion is* _____ .

a) force *b) torque* *c) energy* *d) momentum*

(Ans. : b)

Q.39 *Force system acting on a particle can produce* _____.

a) translational motion *b) rotational motion*

c) deformation *d) all the above*

(Ans. : d)

Q.40 *If a force acting on a rigid body is shifted from one point to another on its line of action, its _____ remains unchanged.*

a) velocity *b) linear acceleration*

c) linear acceleration and angular acceleration

d) angular velocity

(Ans. : c)

Q.41 *Collinear forces are _____ .*

a) parallel to each other *b) concurrent at a point*

c) acting in parallel planes *d) acting on the same line*

(Ans. : d)

Q.42 *Force can be resolved into _____.*

a) two mutually perpendicular components

b) components along any two directions in the plane

c) two parallel components *d) any of the above*

(Ans. : d)

Q.43 *Moment of force is a _____.*

a) scalar *b) vector* *c) dimensionless constant* *d) none of the above*

(Ans. : b)

Q.44 *If moment of force about a certain point A is zero, the line of action of force _____.*

a) does not pass through A *b) passes through A*

c) is at maximum distance from A *d) none of the above*

(Ans. : b)

Q.45 *Anti-clockwise moments in the plane are generally considered positive _____.*

a) just as a convention *b) for convenience*

c) as they point in +z direction *d) none of the above*

(Ans. : c)

Q.46 *To define a force completely, the following characteristics should be specified.*

a) Magnitude and direction *b) Point of application*

c) Line of action *d) All of these* **VTU : Aug.-10**

(Ans. : d)

Q.47 *The net force that results from a number of individual forces acting on an object is the vector sum of the individual forces. This is termed as the _____.*

a) principle of superposition *b) principle of transmissibility*

c) moment of forces *d) principle of physical independence* **VTU : Aug.-10**

(Ans. : a)

Q.48 *Forces co-exist on a plane and all the forces act-helter-skelter over the body. These are _____.*

a) Coplanar non-concurrent forces *b) Coplanar concurrent forces*

c) Coplanar parallel forces *d) Non-coplanar non-concurrent forces*

VTU : Aug.-10, 11

(Ans. : a)

Q.49 *When trying to turn a key into a lock, following is applied _____.*

a) Coplanar forces *b) Moment* *c) Lever* *d) Couple* **VTU : Feb.-11**

(Ans. : d)

Q.50 *Moment of a force _____.*

a) occurs about a point *b) measures the capacity to do useful work*

c) occurs when bodies are in motion

d) measures the ability to produce turning about axes **VTU : Feb.-11**

(Ans. : d)

Q.51 *When more than three concurrent forces are in equilibrium, select the condition that is satisfied.*

a) All the forces must have equal magnitude.

b) Polygon representing the forces will not close.

c) The last side of the polygon will represent the resultant.

d) Polygon representing the forces will close. **VTU : Aug.-11**

(Ans. : d)

Q.52 *The moment of a force about any point is numerically equal to _____ times the area of the triangle whose base is the line representing the force and vertex is the point about which the moment is taken _____.*

a) half *b) same* *c) twice* *d) thrice* **VTU : Aug.-11**

(Ans. : c)

Q.53 *Two forces having the same line of action are called _____ .*

a) coplanar parallel forces *b) non coplanar concurrent forces.*

c) coplanar non concurrent forces *d) collinear forces* **VTU : Dec.-11, Jan-13**

(Ans. : d)

Q.54 *The magnitude of the moment is zero, when the force is applied ___ the lever.*

a) perpendicular to *b) inline with*

c) at any angle to *d) at 60° to* **VTU : Dec.-11**

(Ans. : b)

Q.55 *Following is the unit of moment of a force _____ .*

a) N *b) Nm^2* *c) N^2m* *d) Nm*

VTU : Dec.-11

(Ans. : d)

Q.56 *If two forces are parallel, then they cannot be _____ .*

a) coplanar *b) concurrent* *c) non coplanar* *d) non concurrent*

VTU : Dec.-11

(Ans. : b)

Q.57 *A block of weight W rests on an inclined plane of inclination θ with the horizontal. The magnitude of the component of weight parallel to inclined plane is _____ .*

a) W cosθ *b) W sinθ* *c) W* *d) W tanθ*

(Ans. : b)

Q.58 *A block of weight W rests on an inclined plane of inclination θ with the horizontal. The magnitude of the component of weight perpendicular to inclined plane is _____ .*

a) W cosθ *b) W sinθ* *c) W* *d) W tanθ*

(Ans. : a)

Q.59 *A block of weight W rests on an inclined plane of inclination θ with the horizontal. The magnitude of the horizontal component of weight is _____ .*

a) W cosθ *b) W sinθ* *c) W* *d) 0*

(Ans. : d)

Q.60 *A block of weight W rests on an inclined plane of inclination θ with the horizontal. The magnitude of the vertical component of weight is _____ .*

a) W cosθ *b) W sinθ* *c) W* *d) 0*

(Ans. : c)

Q.61 *The X component of a vector changes when _____ .*

a) moved parallel to itself in the plane *b) moved along its line of action*

c) rotated *d) none of the above*

(Ans. : c)

Q.62 *The Y component of a vector changes when _____ .*

a) moved parallel to itself in the plane *b) moved along its line of action*

c) rotated *d) none of the above*

(Ans. : c)

Q.63 *Which of the following remains unaffected if force is treated as a free vector ?*

a) Translational motion *b) Rotational motion*

c) Deformation *d) All the above*

(Ans. : a)

Q.64 *Which of the following remains unaffected if force is treated as a sliding vector ?*

a) Translational motion *b) Rotational motion*

c) Both a) and b) *d) Deformation*

(Ans. : c)

Q.65 *Which of the following remains unaffected if force is treated as a fixed vector ?*

a) Translational motion *b) Rotational motion*

c) Deformation *d) All the above*

(Ans. : d)

Q.66 *The physical quantity that is applied to open a door is _____ .*

a) force *b) moment* *c) couple* *d) none of the above*

(Ans. : b)

Q.67 *The physical quantity that is applied to open a water tap is _____ .*

a) force *b) moment* *c) couple* *d) none of the above*

(Ans. : c)

Q.68 *Component of a force at right angles to its line of action is _____ .*

a) zero *b) positive* *c) negative* *d) none of the above*

VTU : Aug.-09, June-12, Dec.-13

(Ans. : a)

Q.69 *The moment of force about a moment centre lying on its line of action is _____.*

a) maximum *b) minimum* *c) zero* *d) none of these*

VTU : Dec.-13

(Ans. : c)

Q.70 *Two equal and opposite forces separated by a distance will produce _____.*

a) translation *b) rotation*

c) both translation and rotation *d) none of these*

VTU : Dec.-13

(Ans. : b)

Q.71 *Moment of force will produce _____.*

a) translation *b) rotation* *c) both translation and rotation*

d) none of these

VTU : Dec.-13

(Ans. : b)

❑❑❑

Notes

4 Resultant of Coplanar Force System

Chapter at a Glance

- The concurrent force systems can be reduced to a single force called the resultant force.
- Non-concurrent force systems can be reduced to either a single resultant force with a line of action specified with respect to some point or a single resultant force and a couple moment.
- Resultant of two concurrent forces can be obtained using resolution and composition in cartesian system, triangle law and parallelogram law.
- The force which has same magnitude as resultant force but opposite direction is called **equilibrant.**
- For resolution and composition in cartesian system,

$$R_x = \sum F_x$$

$$R_y = \sum F_y$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\theta = \tan^{-1}\left(\frac{|R_y|}{|R_x|}\right)$$

Sign of R_x and R_y is used to define the quadrant for resultant.

- For parallelogram law,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} \quad \text{and} \quad \tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

where R is the magnitude of resultant and θ is the angle made by R with P.

- Varignon's theorem states that the algebraic sum of moments due to all forces acting on an object about any point is equal to the moment of their resultant about the same point.

Important Theory Questions and Answers

Q.1 *State and prove Varignon's theorem.*

VTU : Feb.-08, 09, Aug.-08, Marks 5, Aug.-10, Marks 6, Jan.-13, June-13

Ans. : Statement : Varignon's theorem states that the algebraic sum of moments due to all forces acting on an object about any point is equal to the moment of their resultant about the same point.

Proof : Consider two forces F_1 and F_2 making angles θ_1 and θ_2 respectively with x-axis and concurrent at the origin as shown in Fig. 4.1. Let their resultant R make angle θ with x-axis. The perpendicular distances of F_1, F_2 and R from point A are d_1, d_2 and d respectively. The distances d_1, d_2 and d can be expressed in terms of OA as follows :

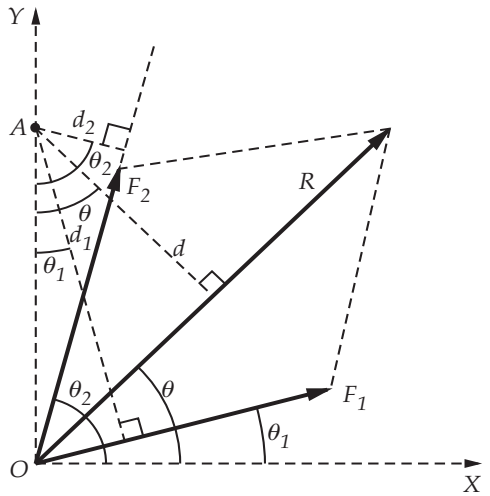


Fig. 4.1

$$d_1 = OA \cos \theta_1$$

$$d_2 = OA \cos \theta_2$$

$$d = OA \cos \theta$$

The moment of R about A is

$$M_R = R\, d$$

$$= R\,(OA \cos \theta)$$

$$\therefore \quad M_R = OA\,(R \cos \theta)$$

The x-component of resultant is

$$R_x = R \cos\theta$$

$$\therefore \quad M_R = OA\, R_x \qquad \ldots (4.1)$$

The moment of F_1 about A is

$$M_1 = F_1\, d_1$$

$$= F_1 (OA \cos \theta_1)$$

$$\therefore \quad M_1 = OA\,(F_1 \cos\theta_1)$$

But $F_1 \cos\theta_1 = F_{1x}$ is the x-component of F_1

$$\therefore \quad M_1 = OA\, F_{1x} \qquad \ldots (4.2)$$

The moment of F_2 about A is

$$M_2 = F_2\, d_2 = F_2\,(OA \cos \theta_2)$$

$$\therefore \quad M_2 = OA\,(F_2 \cos\theta_2)$$

But $F_2 \cos\theta_2 = F_{2x}$ is the x-component of F_2

$\therefore \quad M_2 = OA\ F_{2x}$ ... (4.3)

Adding equations (4.2) and (4.3)

$$M_1 + M_2 = OA\ F_{1x} + OA\ F_{2x}$$

$\therefore \quad M_1 + M_2 = OA\,(F_{1x} + F_{2x})$

But $F_{1x} + F_{2x} = R_x$ is the x-component of resultant

$\therefore \quad M_1 + M_2 = OA \cdot R_x$

$\therefore$ from equation (4.1),

$$M_1 + M_2 = M_R$$

Thus the algebraic sum of moments due to F_1 and F_2 about A is equal to the moment of their resultant about A.

Q.2 *Differentiate between the resultant and the equilibrant.* **VTU : Feb.-11, Marks 4**

Ans. :

1) Resultant produces the same effect as that produced by a given system of forces whereas equilibrant produces the opposite effect.
2) Resultant force and equilibrant force have opposite directions.
3) Resultant tries to produce motion of the object whereas equilibrant keeps the object in equilibrium.

Q.3 *Explain equivalent force-couple system.* **VTU : June-12, Marks 4**

Ans. : Force acting on an object produces translation and rotation of the object. The translational effects are represented by the magnitude and direction of the force whereas the rotational effects are represented by the moment of force. Hence a force can be replaced by a force and couple as shown in Fig. 4.2. Consider a force F acting at point A as shown in Fig. 4.2 (a). Add two forces of magnitudes $\vec{F}$ and $-\vec{F}$. At point O as shown in Fig. 4.2 (b) which are parallel to the force $\vec{F}$ at A. The force $\vec{F}$ at A and the force $-\vec{F}$ at O form a couple with moment Fd about any point in the plane. Thus the force $\vec{F}$ at A is replaced by a force $\vec{F}$ at O and a couple of moment Fd as shown in Fig. 4.2 (c). This procedure can be extended to a system of non-concurrent forces which can be reduced to single resultant force and a couple.

Conversely, a force-couple system can be combined to a single force acting at a distance 'd' from a given point O.

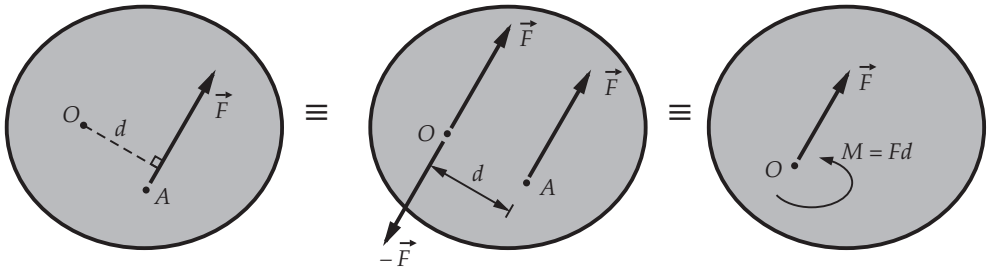


Fig. 4.2

Method for Solving Problems on Resolution and Composition in Cartesian Coordinates

- Resolve each force into x and y component.
- Use

$$R_x = \sum F_x$$
$$R_y = \sum F_y$$
$$R = \sqrt{R_x^2 + R_y^2}$$
$$\theta = \tan^{-1}\left(\frac{|R_y|}{|R_x|}\right)$$

Use sign of R_x and R_y to define the quadrant for resultant.

- If equilibrant is to be calculated, find R and θ as above. The equilibrant will have the same magnitude as R. For its direction, write the same value of θ with opposite quadrant. For example, if resultant is in first quadrant, equilibrant will be in third quadrant.

Method for Solving Problems on Triangle Law

- Draw one of the two forces from the tip of the other. Then complete the triangle by drawing the third side. The third side drawn from the starting point of first force to the end point of the second force defines the resultant in magnitude and direction.
- Then use sine and cosine rules.
- Triangle law can be used for composition and resolution of forces along any two directions in the plane.

Method for Solving Problems on Parallelogram Law

- Assume magnitude of one of the forces to be P and the magnitude of the other to be Q. Assume the angle between the two forces to be α.

- Draw both the forces from the same point.
- Complete the parallelogram by drawing lines parallel to the two forces.
- Draw the diagonal from the common point which represents the resultant in magnitude and direction. Then use

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} \quad \text{and} \quad \tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

where R is the magnitude of resultant and θ is the angle made by R with P.

- Parallelogram law can be used for composition and resolution of forces along any two directions in the plane.

Method for Solving Problems on Finding Resultant of Non - Concurrent Force System

I) Find magnitude and direction of resultant force using rectangular components :

$$R_x = \sum F_x$$

$$R_y = \sum F_y$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\theta = \tan^{-1}\left(\frac{|R_y|}{|R_x|}\right)$$

II) a) If $R = 0$, the resultant can be a moment. Find this resultant moment by taking moment of all the given forces about any convenient point in the plane.

Equilibrant will be a moment having same magnitude as resultant moment but opposite sense of rotation.

b) If $R \neq 0$, the given force system can be reduced to a force-couple system. The moment of couple will be the moment of all the given forces about some given point. Thus a force system can be reduced to a force-couple system as shown in Fig. 4.3.

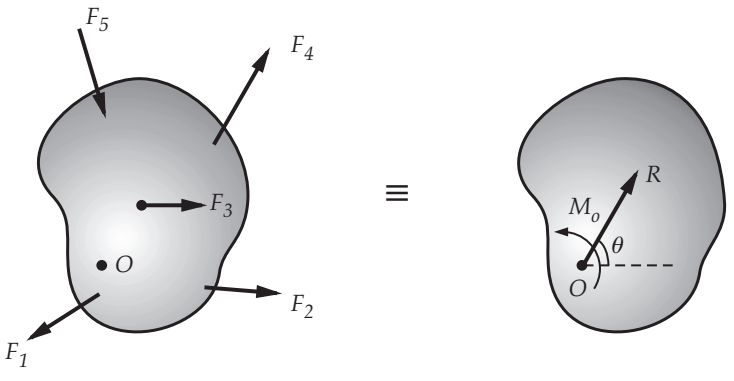


Fig. 4.3

c) If $R \neq 0$, the line of action of resultant force can be specified in the following three ways :

i) Line of action as defined by perpendicular distance 'd' from a specifed point can be obtained using Varignon's theorem.

$$F_1 d_1 + F_2 d_2 + ... = Rd. \quad ... (4.4)$$

If L.H.S is positive, the value of d is also positive representing anticlockwise moment of resultant. A negative value of d will represent clockwise moment of resultant. The resultant can then be represented on a sketch as shown in Fig. 4.4.

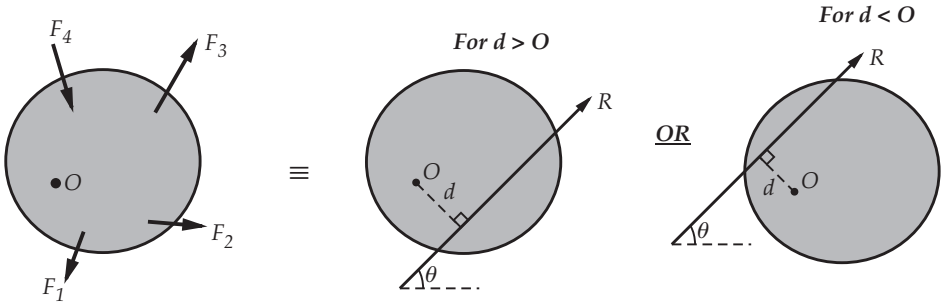


Fig. 4.4

ii) Line of action as defined by x or y intercept :

To find x-intercept, plot the resultant obtained in step I on X-axis at a distance 'x' as shown in Fig. 4.5 and use Varignon's theorem about O :

$$F_1 d_1 + F_2 d_2 + ... = R_y x$$

The R.H.S. representing moment of resultant is written by resolving R.

To find y-intercept, plot the resultant on Y-axis at a distance 'y' as shown in Fig. 4.5 and use Varignon's theorem about O :

$$F_1 d_1 + F_2 d_2 + ... = -R_x y$$

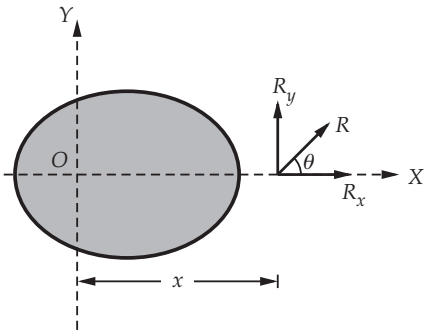


Fig. 4.5

iii) Line of action as defined by point of application of resultant force on any inclined line :

Plot resultant on the inclined line at some distance 'd', as shown in Fig. 4.6 and use Varignon's theorem to determine 'd'.

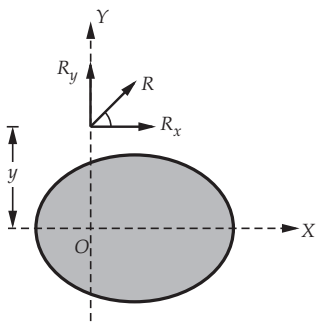


Fig. 4.6

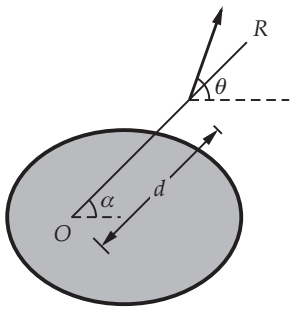


Fig. 4.7

Important Solved Examples

Example 4.1 *The sum of two concurrent forces P and Q is 500 N and their resultant is 400 N. If the resultant is perpendicular to P, find P, Q and angle between P and Q.*

VTU : Feb.-11, Marks 6

Solution : Here we use parallelogram law.

Let α be the angle between P and Q.

Given $P + Q = 500$ ... (1)

$R = 400\ N$ and $\theta = 90^o$

$$\tan\theta = \frac{Q \sin\alpha}{P + Q\cos\alpha}$$

$$\tan 90 = \frac{Q \sin\alpha}{P + Q\cos\alpha}$$

$$\therefore \quad \infty = \frac{Q \sin\alpha}{P + Q\cos\alpha}$$

$$\therefore \quad P + Q\cos\alpha = 0$$

$$\therefore \quad Q\cos\alpha = -P \quad \text{... (2)}$$

$$R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

Substitute from equation (2)

$$400^2 = P^2 + Q^2 + 2P(-P)$$

$$\therefore \quad Q^2 - P^2 = 400^2$$

$$(Q - P)(Q + P) = 400^2$$

$$(Q - P)(500) = 400^2$$

$$\therefore \quad Q - P = 320 \quad \text{... (3)}$$

From equations (1) and (3),

$$P = 90\ N$$

$$Q = 410\ N$$

From equation (2),

$$410 \cos\alpha = -90$$

$$\therefore \quad \alpha = 102.68^{o}$$

Alternatively, the problem can be solved using rectangular components. Consider one of the forces, say P to be along the X-axis as shown in Fig. 4.8.

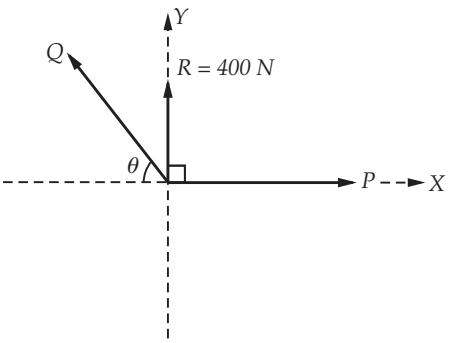


Fig. 4.8

Let θ be angle made by Q with negative X-axis.

$$R_x = \sum F_x$$

$$0 = -Q\cos\theta + P$$

$$\therefore \quad Q\cos\theta = P \qquad \text{... (4)}$$

$$R_y = \sum F_y$$

$$400 = Q\ sin\ \theta \qquad \text{... (5)}$$

Given $P + Q = 500$ ... (6)

From equation (4),

$$\cos\theta = \frac{P}{Q}$$

$$\therefore \quad \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{P^2}{Q^2}}$$

Substitute in equation (5),

$$400 = Q\sqrt{1-\frac{P^2}{Q^2}}$$

$$400^2 = Q^2\left(1-\frac{P^2}{Q^2}\right)$$

$$\therefore \quad Q^2 - P^2 = 400^2$$

$$(Q-P)(Q+P) = 400^2$$

$$(Q-P)(500) = 400^2$$

$$\therefore \quad Q - P = 320 \quad \ldots (7)$$

Solving equations (6) and (7),

$P = 90\ N, \quad Q = 410\ N$

$$\cos\theta = \frac{P}{Q} = \frac{90}{410}$$

$$\therefore \quad \theta = 77.32^o$$

Angle between P and $Q = 180 - 77.32 = 102.68^o$

Example 4.2 *Two forces acting on a body are 500 N and 1000 N as shown in Fig. 4.10. Determine the third force F such that the resultant of all the three forces is 1000 N directed at 45° to the x-axis.*

VTU : Aug.-11, Dec.-13, Marks 6

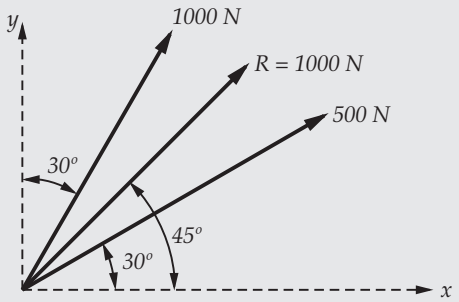


Fig. 4.9

Solution : Let the x and y components of the third force F be F_x and F_y.

$$R_x = \sum F_x$$

$$\therefore \quad 1000\ cos\ 45 = 500\ cos\ 30 + 1000\ sin\ 30 + F_x$$

$$\therefore \quad F_x = -225.9\ N$$

$$R_y = \sum F_y$$

$$1000\ sin\ 45 = 500\ sin\ 30 + 1000\ cos\ 30 + F_y$$

$$\therefore \quad F_y = -408.92\ N$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{225.9^2 + 408.92^2}$$

$$\therefore \quad \boxed{F = 467.17\ N}$$

$$\theta = tan^{-1}\left(\frac{|F_y|}{|F_x|}\right) = tan^{-1}\left(\frac{408.92}{225.9}\right)$$

$$\therefore \quad \boxed{\theta = 61.08^o}$$

Example 4.3 *Determine the magnitude and direction of the resultant for the system of forces shown in Fig. 4.10. Use classical method.*

VTU : June-13, Marks 10

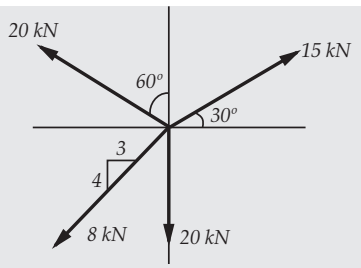


Fig. 4.10

Solution

$$R_x = \sum F_x = 15\cos 30 - 20\sin 60 - 8\times\frac{3}{5} = -9.13 \text{ kN}$$

$$R_y = \sum F_y = 15\sin 30 + 20\cos 60 - 8\times\frac{4}{5} - 20 = -8.9 \text{ kN}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{9.13^2 + 8.9^2}$$

$\therefore$ $\boxed{R = 12.75 \text{ kN}}$

$$\theta = \tan^{-1}\left(\frac{|R_y|}{|R_x|}\right) = \tan^{-1}\left(\frac{8.9}{9.13}\right)$$

$\therefore$ $\boxed{\theta = 44.27^\circ}$

Example 4.4 *Determine the magnitude, direction of the resultant force for the force system shown in Fig. 4.11. Locate the resultant force with respect to point 'D'.*

VTU : Feb.-07, Marks 8; Feb.-09, Marks 10

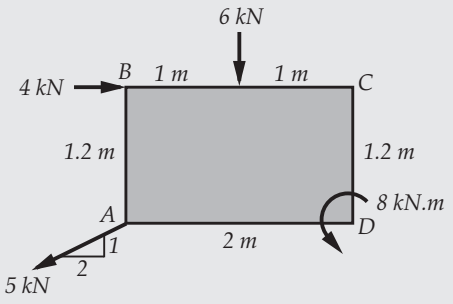


Fig. 4.11

Solution : The angle of *5 kN* force at *A* with horizontal is

$$\tan^{-1}\left(\frac{1}{2}\right) = 26.565^\circ$$

$$R_x = \sum F_x = 4 - 5\cos 26.565$$

$\therefore$ $R_x = -0.472 \text{ kN}$

$$R_y = \sum F_y = -6 - 5\sin 26.565 = -8.236 \text{ kN}$$

$\therefore$ $R = \sqrt{R_x^2 + R_y^2} = \sqrt{0.472^2 + 8.236^2}$

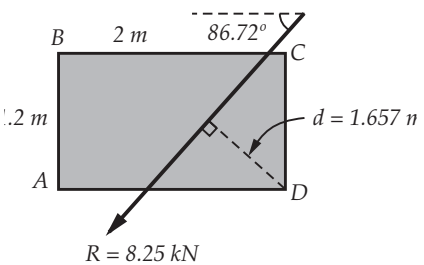


Fig. 4.11 (a)

$\therefore$ $R = 8.25\ kN$

$$\theta = \tan^{-1}\left(\frac{|R_y|}{|R_x|}\right) = \tan^{-1}\left(\frac{8.236}{0.472}\right)$$

$\therefore$ $\theta = 86.72^o$

Using Varignon's theorem at D.

$$8 - (4)\ (1.2) + (6)\ (1) + (5\ sin\ 26.565)\ (2) = (8.25)\ (d)$$

$\therefore$ $d = 1.657\ m$

Positive value of d indicates anticlockwise moment of resultant about D. Hence the position of resultant will be as shown in Fig. 4.11 (a).

Example 4.5 *Replace the horizontal 600 N force acting on the lever as shown in Fig. 4.12 by an equivalent system consisting of a force and a couple at O.*

VTU : Feb.-09, Marks 5

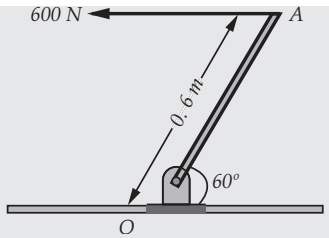


Fig. 4.12

Solution : As there is a single force,

$$R = 600\ N \leftarrow$$

It's moment about O is

$$M_O = (600)(0.6\ sin\ 60)$$

$\therefore$ $M_O = 311.77\ Nm$

Fig. 4.12 (a)

The equivalent system at O is as shown in Fig. 4.12 (a).

Example 4.6 *Replace the force system shown in Fig. 4.13 by a single force passing through A and moment of a couple.*

VTU : Dec.-11, Marks 6

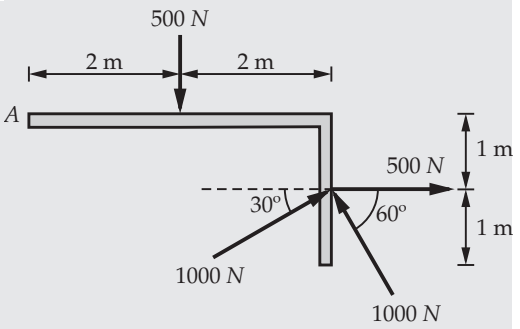


Fig. 4.13

Solution : $R_x = \sum F_x = 500 + 1000\ cos\ 30 - 1000\ cos\ 60 = 866.025\ N$

$$R_y = \sum F_y = -500 + 1000 \sin 30 + 1000 \sin 60 = 866.025 \text{ N}$$

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{866.025^2 + 866.025^2}$$

$$\therefore \quad R = 1224.74 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{|R_y|}{|R_x|}\right) = \tan^{-1}\left(\frac{866.025}{866.025}\right)$$

$$\therefore \quad \theta = 45^o$$

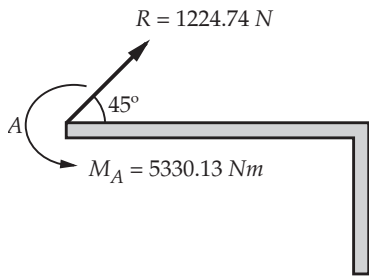


Fig. 4.13 (a)

$$M_A = -(500)(2) + (500)(1) + (1000 \cos 30)(1) + (1000 \sin 30)(4) - (1000 \cos 60)(1) + (1000 \sin 60)(4)$$

$$\therefore \quad M_A = 5330.13 \text{ Nm}$$

The force-couple system at A is as shown in Fig. 4.13 (a).

Important Multiple Choice Questions

Q.1 *The technique of finding the resultant of a system of forces is called* _____.

a) resultant *b) resolution* *c) composition* *d) none of the above*

VTU : Feb.-09,13, June-12,13

(Ans. : c)

Q.2 *Equilibrant is nothing but a resultant* _____ .

a) equal in magnitude and in the same direction

b) equal in magnitude but opposite in direction

c) not equal in magnitude but in the same direction

d) not equal in magnitude and opposite in direction

VTU : Feb.-09, June-13

(Ans. : b)

Q.3 *If two forces P and Q (P > Q) act on the same straight line but in opposite direction their resultant is* _____ .

a) $P + Q$ *b)* $\frac{P}{Q}$ *c)* $Q - P$ *d)* $P - Q$

VTU : Feb.-09, Aug.-10, June-12, 13

(Ans. : d)

Q.4 *In coplanar concurrent force system if* $\sum H = 0$, *then the resultant is* _____ .

a) horizontal *b) vertical* *c) moment* *d) none of the above*

VTU : Feb.-09, June-12, June-13

(Ans. : b)

Q.5 *If two concurrent forces each of magnitude P act at right angles to each other, their resultant is _____ .*

a) 2P *b) P* *c)* $P\sqrt{2}$ *d)* $2\sqrt{P}$

VTU : Aug.-09

(Ans. : c)

Q.6 *The resultant force of two concurrent forces becomes maximum and minimum if angle between them is _____ .*

a) 0^o *and* 180^o *b)* 0^o *and* 90^o *c)* 90^o *and* 0^o *d) none*

VTU : Aug.-09, Jan-13

(Ans. : a)

Q.7 *A rigid body acted upon by coplanar non-concurrent force system has _____ .*

a) both translatory and rotary motion

b) translatory motion in one direction and rotary motion about itself

c) under rest completely *d) All the above*

VTU : Aug.-09

(Ans. : d)

Q.8 *The single force which will have the same effect as the system of force is _____ .*

a) couple *b) resultant* *c) moment* *d) force*

VTU : Feb.-10

(Ans. : b)

Q.9 *If two force M and N (M > N) act on the same straight line but in opposite direction their resultant is _____ .*

a) $M + N$ *b)* $\frac{M}{N}$ *c)* $N - M$ *d)* $M - N$

VTU : Feb.-10

(Ans. : d)

Q.10 *If the resultant of coplanar concurrent force system acts along horizontal x-axis, then _____ .*

a) $\sum F_x = 0$ *b)* $\sum F_x = R$ *c)* $\sum F_y = R$ *d) none of these*

VTU : Feb.-10

(Ans. : b)

Q.11 *The resultant of force system shown in Fig. 4.1 is _____ .*

a) 65 N *b) 40 N*

c) 76.32 N *d) 0*

(Ans. : c)

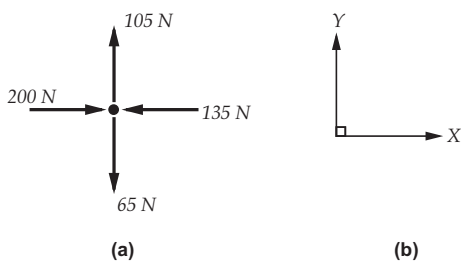


Fig. 4.1

Q.12 *Varignon's theorem is not applicable for forces which are _____.*

a) coplanar non-concurrent *b) non-coplanar non-concurrent*

c) concurrent *d) parallel* **VTU : Feb.-11**

(Ans. : c)

Q.13 *The magnitudes of two given forces are 40 N and 60 N. Which of the following cannot be their resultant ?* **VTU : Dec.-11**

a) 20 N *b) 30 N* *c) 40* *d) 120 N*

(Ans. : (d) The range of magnitude of resultant 'R' is $60 - 40 \leq R \leq 60 + 40$. i.e. $20\ N \leq R \leq 100\ N$)

Q.14 *The magnitudes of two given forces are 30 N and 40 N. Which of the following can be the magnitude of their resultant ?*

a) 0 N *b) 5 N* *c) 10 N* *d) 100 N*

(Ans. : c)

Q.15 *If magnitude of the resultant of two forces of magnitudes p and p is p, the angle between the two forces is _____.*

a) 60^o *b) 30^o* *c) 45^o* *d) 120^o*

(Ans. : (d) The triangle formed by the forces will be an equilateral triangle)

Q.16 *If magnitude of resultant of two forces of magnitudes P and $\sqrt{2}$ P is P, the angle between the two forces is _____.*

a) 135^o *b) 90^o* *c) 45^o* *d) 30^o*

VTU : Feb.-11

(Ans. : (a) The triangle formed will be a right angled triangle)

Q.17 *Two forces of magnitudes $F_1 = P$ and $F_2 = P$ have a resultant of magnitude $R = \sqrt{2}\,P$. The angles made by F_1, F_2 and R with positive x-axis can be _____.*

a) 30^o, 60^o, 90^o *b) 45^o, 45^o, 90^o* *c) 45^o, 90^o, 45^o* *d) 45^o, 135^o, 90^o*

(Ans. : d) The triangle will be a right angle triangle

Q.18 *The angle between two forces of magnitude 100 N each is 120^o. The magnitude of their resultant is _____.*

a) 0 N *b) 100 N* *c) 120 N* *d) 150 N*

(Ans. : b)

Q.19 *The maximum magnitude of resultant of two forces of magnitudes 100 N and 150 N is _____.*

a) 100 N *b) 250 N* *c) 350 N* *d) 125 N*

(Ans. : b)

Q.20 *The minimum magnitude of resultant of two forces of magnitudes 100 N and 150 N is _____.*

a) 100 N *b) 150 N* *c) 50 N* *d) 0*

(Ans. : c)

Q.21 *Two forces of equal magnitude 'P' are applied on a particle at the origin along positive x and y directions. A third force 'F' is to be applied on the particle. The direction of 'F' to make the resultant of the three forces maximum is _____.*

a) 45° with positive x-axis *b) 135° with positive x-axis*

c) 225° with positive x-axis *d) 60° with positive x-axis*

(Ans. : (a) The third force must be applied in the direction of resultant of the other two forces)

Q.22 *If the resultant in Q.22 is to be made minimum, 'F' has to be applied at _____.*

a) 45° with positive x-axis *b) 135° with positive x-axis*

c) 225° with positive x-axis *d) 60° with positive x-axis*

(Ans. : (c) The third force must be appllied opposite to direction of resultant of the other two forces)

Q.23 *If the angle between two forces each of magnitude P is 50°, then their resultant has magnitude _____.*

a) 2P *b) 2P cos 50* *c) P cos 50* *d) 2P cos 25*

(Ans. : d)

Q.24 *Two forces acting on a particle in opposite direction have a resultant of 5 N. If they act at right angles to each other, their resultant has a magnitude of 25 N. The magnitudes of the two forces are _____.*

a) 10 N, 15 N *b) 15 N, 20 N* *c) 20 N , 25 N* *d) 5 N, 10 N*

(Ans. : b)

Q.25 *Two forces of magnitudes 8 N and 6 N act at a point. If the resultant of the two forces has a magnitude of 10 N, the angle between the two forces is _____.*

a) 90° *b) 60°* *c) 45°* *d) 30°*

(Ans. : a)

Q.26 *Two forces* $\vec{F}_1$ *and* $\vec{F}_2$ *are such that*

$$\left|\vec{F}_1 + \vec{F}_2\right| = \left|\vec{F}_1 - \vec{F}_2\right|$$

Then, the angle between $\vec{F}_1$ *and* $\vec{F}_2$ *will be _____.*

a) 0° *b) 180°* *c) 60°* *d) 90°*

(Ans. : d)

Q.27 *If magnitudes of three forces* $\vec{F}_1, \vec{F}_2$ *and* $\vec{F}_3$ *are 5 N, 12 N and 13 N respectively and* $\vec{F}_1 + \vec{F}_2 = \vec{F}_3$, *then the angle between* $\vec{F}_1$ *and* $\vec{F}_2$ *is _____.*

a) 0° *b) 90°* *c)* $\cos^{-1}\left(\frac{5}{13}\right)$ *d)* $\tan^{-1}\left(\frac{5}{12}\right)$

(Ans. : (b) As $\sqrt{5^2 + 12^2} = 13$, $\vec{F}1$ **and** $\vec{F}2$ **are perpendicular to each other)**

Q.28 *If* $\vec{F}_1 = \vec{F}_2 + \vec{F}_3$ *and magnitudes of* $\vec{F}_1, \vec{F}_2$ *and* $\vec{F}_3$ *are 5 N, 3 N and 4 N respectively, then the angle between* $\vec{F}_1$ *and* $\vec{F}_2$ *is* _____.

a) $\cos^{-1}\left(\frac{4}{5}\right)$ *b)* $\cos^{-1}\left(\frac{3}{5}\right)$ *c)* 90^o *d)* $\sin^{-1}\left(\frac{3}{4}\right)$

(Ans. : b)

Q.29 *The maximum moment of the 40 N force shown in Fig. 4.15 about A is* _____.

a) 160 Nm *b) 120 Nm*

c) 160 Nm *d) 200 Nm*

(Ans. : (d) The moment is maximum when the force is perpendicular to AB)

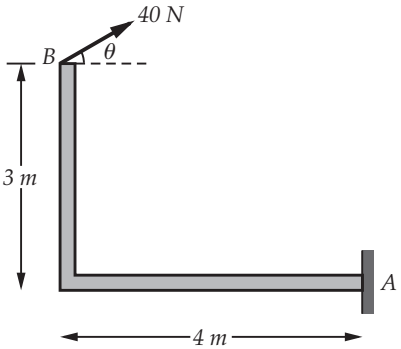


Fig. 4.15

Q.30 *The x-component of force in Fig. 4.15 is* _____ .

a) 0 *b) 40 N*

c) 24 N *d) 32 N.*

(Ans. : c)

Q.31 *The y-component of force in Fig. 4.15 is* _____.

a) 0 N *b) 40 N*

c) 24 N *d) 32 N*

(Ans. : d)

Q.32 *The minimum moment of the 40 N force shown in Fig. 4.15 is* _____.

a) 0 *b) 120 Nm* *c) 160 Nm* *d) 200 Nm*

(Ans. : a)

Q.33 *The x-component of force in Q.33 is* _____.

a) 24 N *b) 32 N* *c) 0* *d) 40 N*

(Ans. : (b) The force will be directed along AB)

Q.34 *The y-component of force in Q.33 is* _____.

a) – 32 N *b) 32 N* *c) 24 N* *d) – 24 N*

(Ans. : d)

Q.35 *A 100 N force is to be applied to a rectangular plate shown in Fig. 4.16 to produce maximum moment about E. The force must be applied at* _____.

a) A *b) B*

c) C *d) D*

(Ans. : (c) The force must be applied at maximum distance from the hinge for maximum moment)

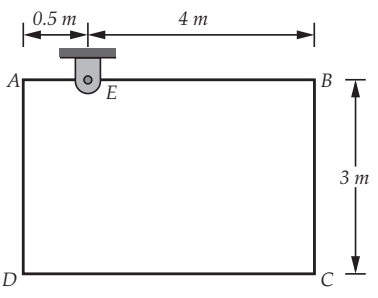


Fig. 4.16

Q.36 *The maximum moment of the force in Q.35 about E is _____.*

a) 500 Nm *b) 300 Nm*

c) 400 Nm *d) 450 Nm*

(Ans. : a)

Q.37 *The magnitude of x-component of force in Q.35 is _____.*

a) 80 N *b) 60 N*

c) 50 N *d) 40 N*

(Ans. : b)

Q.38 *The magnitude of y-component of force in Q.35 is _____.*

a) 80 N *b) 60 N* *c) 50 N*
d) 40 N

(Ans. : a)

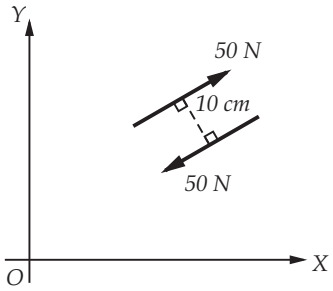


Fig. 4.17

Q.39 *The moment of the two forces shown in Fig. 4.18 about 'O' is _____.*

a) 5 Nm ↻

b) 5 Nm ↺

c) 50 Nm ↻

d) data is insufficient to find the moment.

(Ans. : (a) M = $50 \times \frac{10}{100}$ ↻ = 5 Nm ↻)

Q.40 *The moment of the 20 N force shown in Fig. 4.19 about 'O' is*

a) 4 Nm ↻ *b) 4 Nm* ↺

c) 5 Nm ↻ *d) 5 Nm* ↺

(Ans. : (b) Resolve the force into x and y components to find the moment)

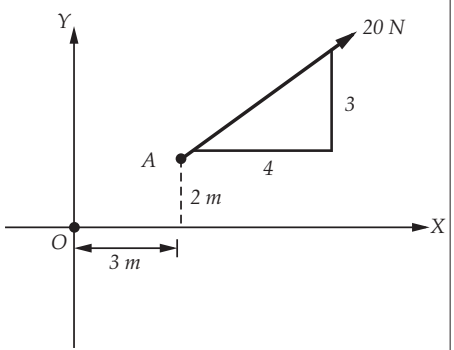


Fig. 4.18

Q.41 *If the resultant of the three forces shown in Fig. 4.6 is horizontal, the angle θ is*

a) 0° *b) 30°*

c) 45° *d) 60°*

(Ans. : (b) As resultant is horizontal)

$\sum F_y = 0 \therefore 200 \sin \theta - 100 = 0 \therefore \theta = 30°$)

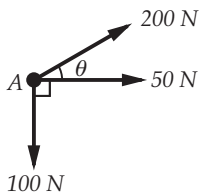


Fig. 4.19

Q.42 *The mathematical statement of triangle law is known as _____.*

a) law of sines *b) law of cosines*

c) polygon law *d) parallelogram law*

(Ans. : a)

Q.43 *The mathematical statement of parallelogram law is known as _____.*

a) law of sines *b) law of cosines* *c) polygon law* *d) triangle law*

(Ans. : b)

Q.44 *In _____ law every vector is drawn from the tip of the previous vector to find resultant.*

a) parallelogram law *b) polygon law*

c) composition and resolution in rectangular components

d) none of the above

(Ans. : b)

Q.45 *In _____ law both vectors are drawn from a common point to find resultant.*

a) parallelogram law *b) polygon law*

c) triangle law *d) none of the above*

(Ans. : a)

Q.46 *The magnitude of resultant of two forces increases when angle between them _____.*

a) increases from 0^o to 90^o *b) increases from 0^o to 180^o*

c) increases from 180^o to 360^o *d) none of the above*

(Ans. : c)

Q.47 *The magnitude of resultant of two forces decreases when angle between them _____.*

a) increases from 0^o to 180^o *b) increases from 180^o to 360^o*

c) increases from 0^o to 360^o *d) none of the above*

(Ans. : a)

Q.48 *Parallelogram law _____.*

a) can be proved mathematically *b) can be derived*

c) can be verified experimentally *d) all the above*

(Ans. : c)

Q.49 *The simplest resultant of a parallel force system is always _____.*

a) a wrench *b) a force* *c) a moment* *d) a force and a moment*

VTU : Aug.-10

(Ans. : d)

Q.50 *The purpose of replacing a number of forces acting on a body simultaneously is _____.*

a) to find resolution *b) to find transmission*

c) to find resultant *d) to find moment of forces*

VTU : Aug.-10

(Ans. : c)

Q.51 *If two forces act at an angle of 120°, the greater force is 50 N and their resultant is perpendicular to the smaller force, the smaller force is _____.*

a) 20 N *b) 25 N* *c) 30 N* *d) 43.33 N* **VTU : Aug.-10**

(Ans. : b)

Q.52 *A single force and a couple acting in the same plane upon a rigid body _____.*

a) balance each other *b) can not balance each other*

c) produce moment of a couple *d) one equivalent.* **VTU : Feb.-11, Jan-13**

(Ans. : b)

Q.53 *Two forces each equal to P/2 act at right angles. Their effect may be neutralized by the third force, acting along their bisector in the opposite direction, with a magnitude of _____.*

a) P *b)* $\sqrt{2P}$ *c)* $-P/2$ *d)* $P/\sqrt{2}$ **VTU : Feb.-11**

(Ans. : d)

Q.54 *In a coplanar concurrent forces system, if* $\Sigma V = 0$*, then the resultant is*

a) horizontal *b) vertical* *c) moment* *d) none of these* **VTU : Feb.-11**

(Ans. : a)

Q.55 *If two equal forces of magnitude 'p' act at an angle '*θ*', their resultant will be _____.*

a) $2p\cos\theta/2$ *b)* $P\tan\theta/2$ *c)* $2p\sin\theta/2$ *d)* $p\cos\theta/2$

VTU : Aug.-11, Jan-13

(Ans. : a)

Q.56 *The simplest resultant of a plane force system is always _____.*

a) a single force *b) a wrench*

c) a single moment *d) a single force or a single moment* **VTU : Aug.-11**

(Ans. : d)

Q.57 *The angle between two forces to make their resultant a minimum and a maximum respectively are : _____.*

a) 0° and 90° *b) 180° and 90°* *c) 90° and 180°* *d) 180° and 0°* **VTU : Aug.-11**

(Ans. : d)

Q.58 *The resultant of two concurrent forces becomes minimum if angle between them is _____ .*

a) zero *b) 180°* *c) 90°* *d) 60°*

VTU : Dec.-11

(Ans. : b)

Q.59 *If two concurrent forces each of magnitude P act at right angles to each other, their resultant is _____ .*

a) 2P *b) Zero* *c) P $\sqrt{2}$* *d) (P/2)*

VTU : Dec.-11

(Ans. : c)

Q.60 *If the magnitude of resultant of two forces, of each magnitude P, is P, then the angle between the two forces is _____.*

a) Zero *b) 45°* *c) 120°* *d) 60°*

VTU : Dec.-11,13

(Ans. : c)

Q.61 *Two forces, each of magnitude 10 N, act on a particle. One of them is directed 30° N of E and the other at 60° W of N. The resultant force has magnitude _____ .*

a) 0 *b) 10 N* *c) 5 N* *d) 20 N*

(Ans. : b)

Q.62 *Two forces, each of magnitude 10 N, act on a particle. One of them is directed 30° N of E and the other at 60° W of N. The resultant force is directed towards _____ .*

a) North *b) East* *c) West* *d) South*

(Ans. : a)

Q.63 *Two forces, each of magnitude 10 N, act on a particle. One of them is directed 30° N of E and the other at 60° W of S. The resultant force has magnitude _____ .*

a) 0 *b) 10 N* *c) 5 N* *d) 20 N*

(Ans. : a)

Q.64 *The minimum magnitude of resultant of three forces, each of magnitude 10 N is _____ .*

a) 0 *b) 10 N* *c) 20 N* *d) 30 N*

(Ans. : a)

Q.65 *The maximum magnitude of resultant of three forces, each of magnitude 10 N is _____ .*

a) 0 *b) 10 N* *c) 20 N* *d) 30 N*

(Ans. : d)

Q.66 *Two equal, opposite and collinear forces acting on a rigid body _____.*

a) form couple *b) have zero resultant*

c) have non-zero resultant *d) none of the above*

(Ans. : b)

Q.67 *The effect of a couple acting on a rigid body can be nullified by applying equal and opposite _____.*

a) Force *b) couple* *c) force and couple* *d) none of the above*

(Ans. : b)

Q.68 *A couple moment of 5 kNm acts on a beam AB at a distance of 2 m from A. The moment about A is _____ kNm.*

a) 10 *b) 0* *c) 5* *d) none of these.*

(Ans. : c)

Q.69 *The force $\vec{F_1}+\vec{F_2}$ bisects the angle between $\vec{F_1}$ and $\vec{F_2}$ if _____.*

a) $|\vec{F_1}| = |\vec{F_2}|$ *b)* $|\vec{F_1}| = 2\,|\vec{F_2}|$ *c)* $|\vec{F_1}| = \frac{1}{2}\,|\vec{F_2}|$ *d) None of the above.*

(Ans. : a)

Q.70 *In non-concurrent force system if* $\sum H = 0$ *and* $\sum V = 0$ *then the resultant is ____.*

a) resultant force *b) force* *c) moment* *d) zero.*

VTU : Feb.-09

(Ans. : c)

Q.71 *The procedure of resolution is _____.*

VTU : June-13

a) To find the resultant of the force system

b) To break up an inclined force into two components

c) To find the equilibriant

d) None of these

(Ans. : b)

Q.72 *In case of coplanar concurrent force the resultant force passes through _____.*

VTU : Dec.-13

a) point of concurrence

b) away from point of concurrence

c) different plane

d) none of these

(Ans. : a)

Q.73 *If* $\sum V = 0$ *and* $\sum H = 0$ *for a coplanar nonconcurrent force system, then it is ____.*

VTU : June-13, Dec.-13

a) equilibrium *b) translation* *c) rotation* *d) none of these*

(Ans. : c)

❑❑❑

Notes

5 Equilibrium

Chapter at a Glance

- A body is said to be in equilibrium provided it remains at rest if initially at rest or has constant velocity if initially it was moving.
- Equilibrant is defined as a force or a moment required to keep an object in equilibrium.
- The conditions for equilibrium of a concurrent for system are

$$\sum F_x = 0$$

$$\sum F_y = 0$$

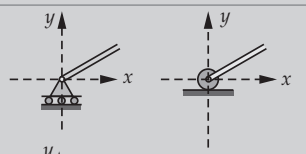
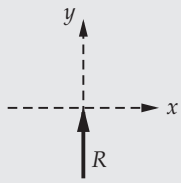
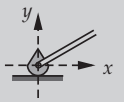
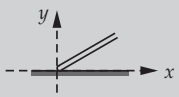
- The conditions for equilibrium of coplanar non-concurrent force systems are

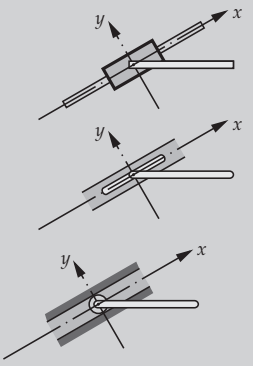
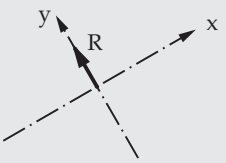
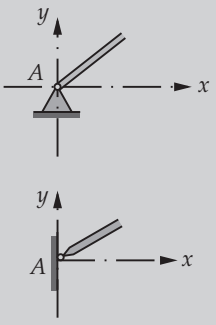
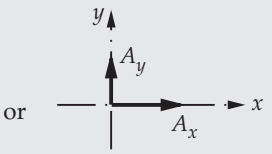
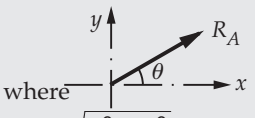
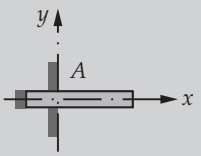
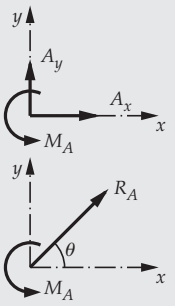
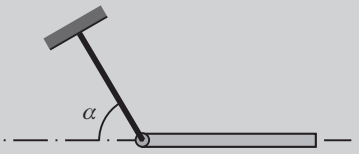
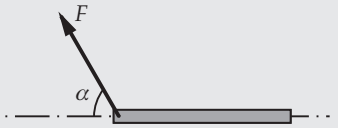
$$\sum F_x = 0$$

$$\sum F_y = 0$$

and $$\sum M = 0$$

- Types of supports and corresponding reactions :

Sr. No.	Type of support	Symbolic representation	Reaction components
1.	a) Roller		
	b) Rocker		Reaction is perpendicular to the surface on which object is resting.
	c) Object on frictionless surface		

2.	a) Collar on frictionless rod. b) Pin in a slot without friction c) Roller in a slot		 Reaction is perpendicular to the rod or the slot.
3.	Pin or hinge		 or  where $R_A = \sqrt{A_x^2 + A_y^2}$ $\theta = tan^{-1} \frac{A_y}{A_x}$
4.	Fixed		or 
5.	Cable		 There is tensile force in the cable.

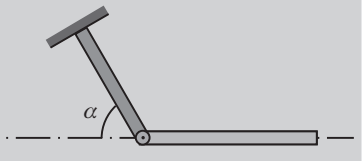
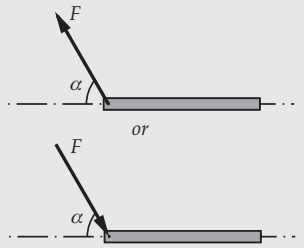
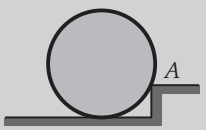
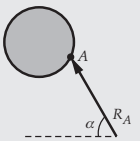
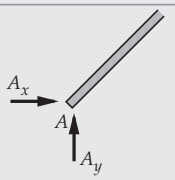
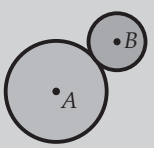
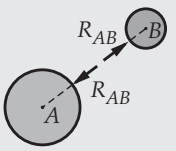
6.	Weightless link		 Link (i.e. rod) can have tensile or compressive force when subjected to forces only at the ends.
7.	Roller at a sharp edge		 The direction of reaction is towards the roller but the angle 'α' depends on the other applied forces.
8.	Corner		
9.	Two rollers		 Reaction is along the line joining the two centres.

Table 5.1 Types of supports and corresponding reactions

- **Lami' Theorem :** For the three concurrent forces F_1, F_2 and F_3 as shown in Fig. 5.1.

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

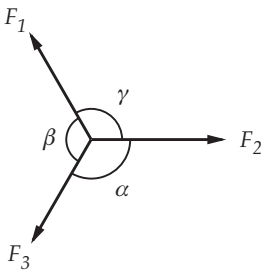


Fig. 5.1

Method for Solving Problems

1. Draw F.B.D. of a point in the system/an object in the system/complete system of objects in equilibrium. Consider all forces acting on the system including support reactions.

2. If forces are concurrent, use

$$\sum F_x = 0, \quad \sum F_y = 0$$

3. If forces are not concurrent, use

$$\sum F_x = 0, \quad \sum F_y = 0 \quad \text{and} \quad \sum M = 0$$

4. For three concurrent forces in equilibrium, use Lami's theorem.
5. For an object hanging freely from a cable, the tension in the cable will be equal to the weight of the object.
6. For equilibrium of a system of more than one objects, draw free body diagrams of all the objects. Start solving with that F.B.D. which has minimum number of unknowns.

Important Theory Questions and Answers

Q.1 *State and prove Lami's theorem.* **VTU : Aug.-07, Dec.-13**

Ans. : Statement : If three concurrent forces are in equilibrium, the ratio of magnitude of any force to the sine of angle between the other two forces is constant.

For the three concurrent forces F_1, F_2 and F_3 as shown in Fig. 5.2.

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma} \qquad \ldots (5.1)$$

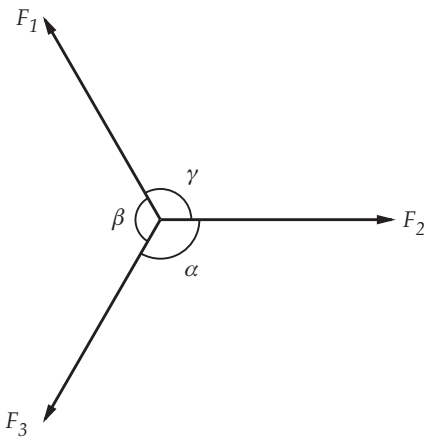


Fig. 5.2

Proof : As the three forces are in equilibrium, they will form a closed triangle when drawn one after the other as shown in Fig. 5.9.2.

Using sine rule,

$$\frac{F_1}{\sin(180-\alpha)} = \frac{F_2}{\sin(180-\beta)} = \frac{F_3}{\sin(180-\gamma)}$$

$$\therefore \quad \frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

Q.2 *Define free body diagram. Describe types of forces acting on a body. Explain them in brief.* **VTU : Aug.-10, 11, Marks 6**

Ans. : It is the isolated diagram of an object/system of objects/any point in the system in which all forces and couple moments acting on it are shown including support reactions.

Types of forces : The two basic types of forces are surface forces and body forces.

Surface forces are those which act on the surface of objects. These forces include the forces that are applied on objects by us, reaction forces which are exerted on the object by its supports and the force of friction which arises due to relative motion between two surfaces in contact.

Body forces are those which act on each and every particle of the object. For example, the gravitational force (i.e, the weight of the object) is a body force.

Forces which act on an object due to their acceleration or deceleration are known as inertia forces. These are pseudo forces. For example, when a vehicle accelerates, we experience a force in the backward direction due to the acceleration of the vehicle.

Important Solved Examples

Example 5.1 *Two spheres each of radius 100 mm and weight 5 kN is in a rectangular box as shown in Fig. 5.3. Calculate the reactions at all the points of contact.*

VTU : Feb.-07, June-13, Marks 10

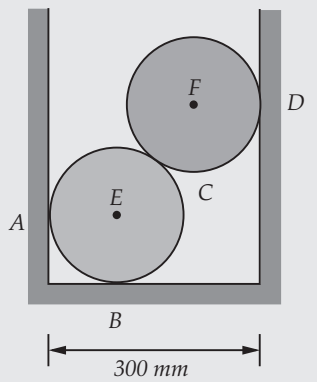


Fig. 5.3

Solution : The reaction between the two spheres is along the line joining their centres. The angle α for this reaction with horizontal can be obtained using constructions shown in Fig. 5.3 (a).

$$\cos\alpha = \frac{100}{200}$$

$$\therefore \quad \alpha = 60^o$$

The free body diagrams of the two spheres are shown in Fig. 5.3 (b).

Using Lami's theorem for sphere F,

$$\frac{R_C}{\sin 90} = \frac{R_D}{\sin 150} = \frac{5}{\sin 120}$$

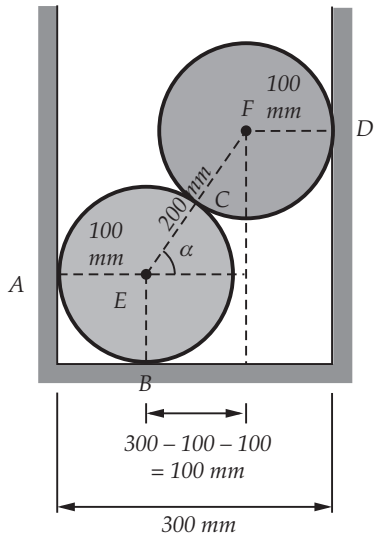


Fig. 5.3 (a)

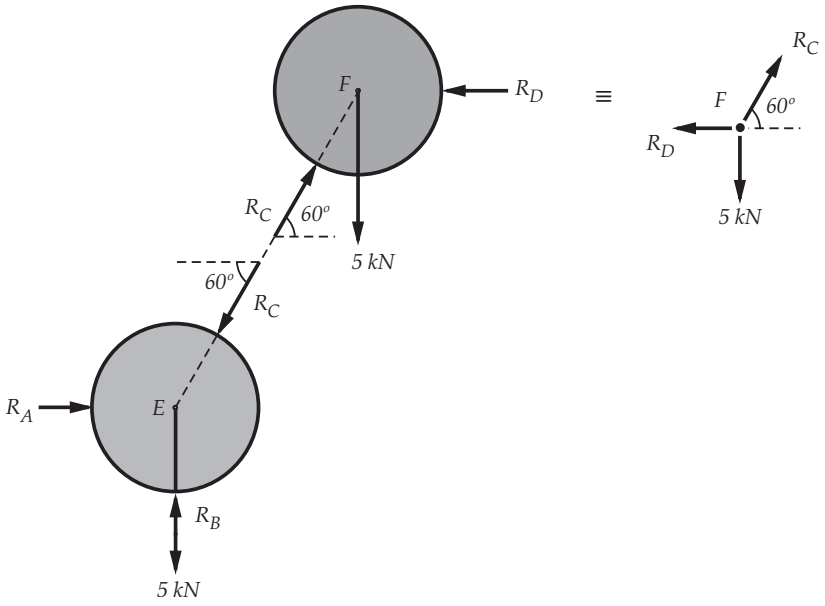


Fig. 5.3 (b)

$\therefore$ $R_C = 5.7735\ kN$

$\therefore$ $R_D = 2.887\ kN \leftarrow$

For F.B.D. of E, $\sum F_x = 0$:

$$R_A - R_C \cos 60 = 0$$

$$R_A = 5.7735 \cos 60$$

$\therefore$ $R_A = 2.887\ kN \rightarrow$

$\sum F_y = 0$:

$$R_B - 5 - R_C \sin 60 = 0$$

$$R_B = 5 + 5.7735 \sin 60$$

$\therefore$ $R_B = 10\ kN \uparrow$

Example 5.2 *Determine the angle θ for the system of strings ABCD in equilibrium as shown in Fig. 5.4.*

VTU : Feb.-09, Aug.-10, Marks 10
June-12,13, Marks 8

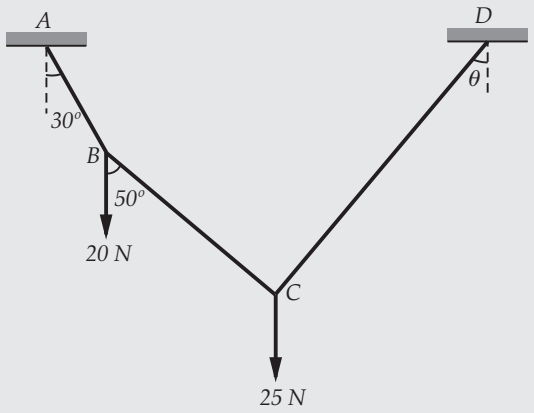


Fig. 5.4

Solution : The free body diagrams of B and C are shown in Fig. 5.4 (a) .

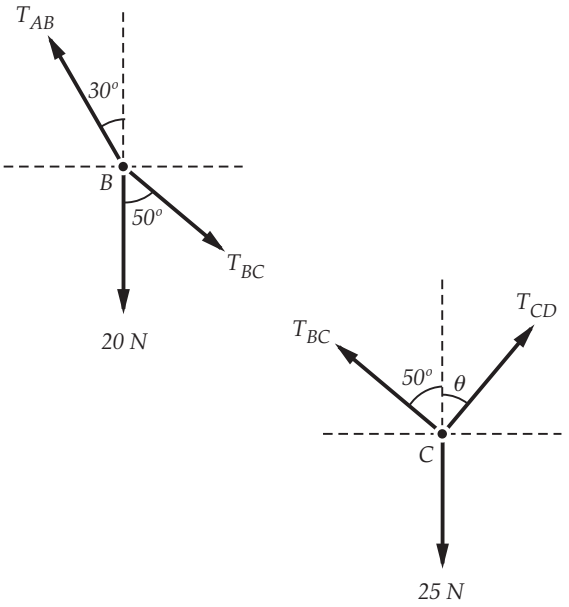


Fig. 5.4 (a)

Using Lami's theorem for B,

$$\frac{T_{AB}}{\sin 50} = \frac{T_{BC}}{\sin 150} = \frac{20}{\sin 160}$$

$\therefore$ $T_{AB} = 44.8\ N$

$T_{BC} = 29.24\ N$

For C ; $\sum F_x = 0$:

$T_{CD} \sin\theta - T_{BC} \sin 50 = 0$

$$\therefore \quad T_{CD} \sin\theta = 29.24 \sin 50 \quad \ldots (1)$$

$\sum F_y = 0$:

$$T_{CD} \cos\theta + T_{BC} \cos 50 - 25 = 0$$

$$T_{CD} \cos\theta = 25 - 29.24 \cos 50 \quad \ldots (2)$$

Dividing equation (1) by (2),

$$\tan\theta = \frac{29.24 \sin 50}{25 - 29.24 \cos 50}$$

$$\therefore \quad \boxed{\theta = 74.52^o}$$

Example 5.3 *Find force 'F' acting on crank for equilibrium and also find support reaction. Refer Fig. 5.5.*

VTU : Dec.-11, Marks 10

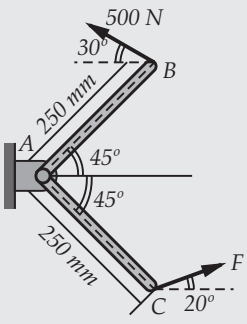


Fig. 5.5

Solution : The FBD of crank is shown in Fig. 5.5 (a).

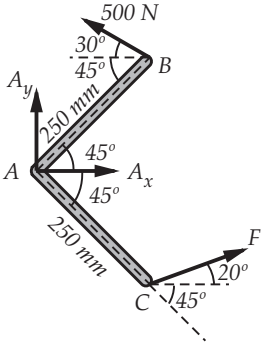


Fig. 5.5 (a)

$\sum M_A = 0$: $(500 \sin 75)(250) + (F \sin 65)(250) = 0$

$$\therefore \quad F = -532.89 \text{ N}$$

$$\therefore \quad F = 532.89 \text{ N}, 20^o \swarrow$$

(Note that the forces have been revolved parallel and perpendicular to the rods AB and AC for the purpose of taking moment)

$\sum F_x = 0$: $A_x - 500 \cos 30 + F \cos 20 = 0$

$$A_x = 500 \cos 30 - (-532.89 \cos 20)$$

$$A_x = 933.77 \text{ N} \rightarrow$$

$\sum F_y = 0$: $A_y + 500 \sin 30 + F \sin 20 = 0$

$$A_y = -500 \sin 30 - (-532.89 \cos 20) = -67.74$$

$$\therefore \quad A_y = 67.74 \text{ N} \downarrow$$

$$R_A = \sqrt{A_x^2 + A_y^2} = \sqrt{933.77^2 + 67.74^2}$$

$\therefore$ $R_A = 936.22$ N

$$\theta = \tan^{-1}\left(\frac{|A_y|}{|A_x|}\right) = \tan^{-1}\left(\frac{67.74}{933.77}\right)$$

$\therefore$ $\theta = 4.15^o$

Example 5.4 *Determine the reactions at the point of contact for the sphere shown in Fig. 5.6.*

VTU : June-13, Marks 6

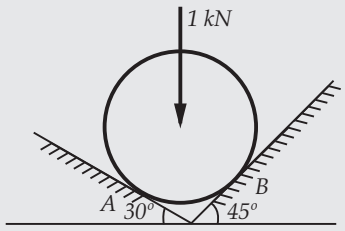


Fig. 5.6

Solution. : The F.B.D. of sphere is shown in Fig. 5.6 (a).

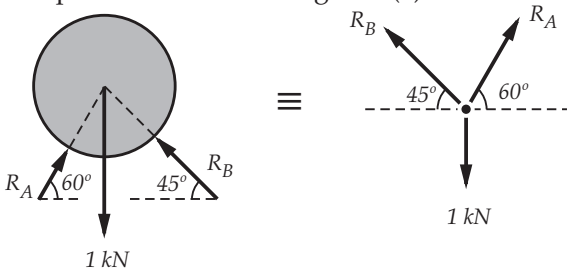


Fig. 5.6 (a)

Using Lami's theorem,

$$\frac{R_A}{\sin 135} = \frac{R_B}{\sin 150} = \frac{1}{\sin 75}$$

$R_A = 0.732$ kN

$R_B = 0.518$ kN

Important Multiple Choice Questions

Q.1 *The necessary condition for equilibrium of a coplanar concurrent force system is that algebraic sum of ___ must be zero.*

a) horizontal and vertical forces *b) moment of forces*

c) horizontal, vertical forces and moments *d) none of the above.*

VTU : Feb.-09, Aug.-10, June-12

(Ans. : a)

Q.2 *The force which is equal and opposite to the resultant is _____.*

a) resultant force b) force c) equilibrant d) none of the above

VTU : Feb.-09, 10, June-12,13, Dec.-13

(Ans. : c)

Q.3 *The procedure of resolution is ____.*

a) to find the resultant of the force system

b) to break up an inclined force into two components

c) to find the equilibrant d) none of the above

VTU : Feb.-09

(Ans. : b)

Q.4 *A rigid body is in equilibrium if the resultant force of concurrent force system is _____.*

a) positive b) negative c) zero d) none of these

VTU : Aug.-09

(Ans. : c)

Q.5 *A system of force that possesses resultant force move in ___.*

a) the direction of line of action of resultant

b) opposite to the direction of line of action of resultant

c) perpendicular to the direction of line of action of resultant

d) none of these

VTU : Aug.-09

(Ans. : a)

Q.6 *Lami's theorem is valid for _____ .*

a) two concurrent forces in equilibrium

b) four concurrent forces in equilibrium

c) three concurrent forces in equilibrium

d) none of the above

VTU : Aug.-09

(Ans. : c)

Q.7 *For a smooth spherical surface reaction acts _____.*

a) horizontal to the plane of contact

b) inclined to the plane of contact

c) perpendicular to the plane of contact

d) none of the above

VTU : Aug.-09,11, Jan-13

(Ans. : c)

Q.8 *Lami's equation can be applied when number of unknown forces are _____.*

a) two b) three

c) five d) none of these

VTU : Feb.-10, Dec.-11, June-12, Jan-13

(Ans. : a)

Q.9 *The necessary condition for equilibrium of coplanar concurrent force system is _____.*

a) $\sum F_y = \sum F_x$ *b)* $\sum F_x = 0, \sum F_y = 0$

c) $\sum m = 0$ *d)* $\sum F_x - \sum F_y = 1$

VTU : Feb.-10

(Ans. : b)

Q.10 *A system that possesses a resultant _____.*

a) will be in equilibrium *b) will be under rest*

c) will not be in equilibrium *c) none of these*

VTU : Feb.-10, Aug.-11

(Ans. : c)

Q.11 *If a body is in equilibrium, we may conclude that _____.*

a) no force is acting on the body

b) the resultant of all the forces acting on it is zero

c) the moments of the forces about any point is zero

d) both (b) and (c)

VTU : Feb.-11, Jan-13

(Ans. : d)

Q.12 *If the sum of all the forces acting on a body is zero, then the body may be in equilibrium provided the forces are _____.*

a) concurrent *b) parallel* *c) like parallel* *c) unlike parallel*

(Ans. : a)

Q.13 *If a body is at rest, it implies that _____.*

a) the forces acting of on it are always zero

b) the resultant of the forces acting on it are zero

c) the moment of the forces acting on it are zero

d) both the resultant force and moment are zero

(Ans. : d)

Q.14 *If a body is in equilibrium then it implies that the body _____.*

a) is at rest

b) is at rest or moving with constant velocity

c) is moving with constant acceleration

d) is oscillating about a fixed point

(Ans. : b)

Q.15 *A free-body diagram is a diagram _____.*

a) drawn by free hand

b) of a body suspended freely in air

c) of a body in vacuum free from any influence from the surroundings

d) drawn by detaching the body from its attachments with the surroundings and replacing the attachments with force vectors

VTU : Jan-13,Feb.-11

(Ans. : d)

Q.16 *The forces shown in the free-body diagram are _____.*

a) the forces exerted by the body under consideration upon the surrounding bodies

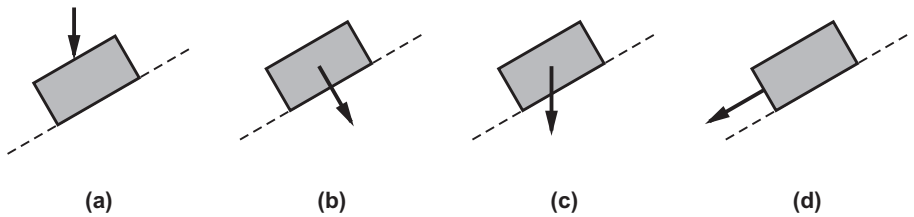
b) the forces exerted by the surrounding bodies upon the body under consideration

c) the internal forces resisting deformation of the body

d) the forces exerted in free space

(Ans. : b)

Q.17 *A body is resting on an inclined plane. Which of the following figure represents the correct weight vector ?*



(Ans. : c)

Q.18 *Three forces acting on a body can keep it in equilibrium, only when they are _____.*

a) collinear *b) coplanar and concurrent*

c) coplanar and parallel *d) coplanar and non-concurrent*

(Ans. : b)

Q.19 *If resultant of a concurrent force system acting on an object is zero, the object _____.*

a) will remain at rest if it was initially at rest

b) moves with constant velocity

c) has zero acceleration

d) any the above

(Ans. : d)

Q.20 *If resultant torque acting on a rigid body is zero, _____.*

a) the rigid body rotates with constant angular velocity

b) the body has constant angular acceleration

c) the angular acceleration increases with time

d) the angular acceleration decreases with time

(Ans. : a)

Q.21 *A particle acted upon by two forces of equal magnitudes is in equilibrium. The angle between the two forces is _____.*

a) 0^o *b) 90^o* *c) 180^o* *d) 45^o*

VTU : Feb.-11, Dec.-11, June-12

(Ans. : c)

Q.22 *If forces F_1 and F_2 act along a straight line and F_3 is inclined at angle θ with F_1, then for equilibrium _____.*

a) $F_3 = 0$ *b) $F_3 = F_1 \cos\theta$* *c) $F_3 = F_1 \sin\theta$* *d) $F_3 = F_2 \sin\theta$*

(Ans. : a)

Q.23 *The normal reaction on the block of mass 'm' lying on a horizontal plane and a force P acting on it at angle 'θ' in fourth quadrant is _____.*

a) mg *b) mg + P* *c) mg + P cos θ* *d) mg + P sin θ*

(Ans. : d)

Q.24 *A mass 'm' is supported by a cable ABC. A horizontal force 'P' is applied at B as shown in Fig. 5.7 so that AB makes angle θ with the vertical. The tensile force in AB will be _____.*

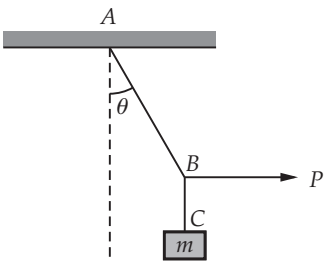


Fig. 5.7

a) P sin θ *b) P cosec θ*

c) P cos θ *d) P sec θ*

(Ans. : b)

Q.25 *The tension in AB in Q.24 can also be written as _____.*

a) mg cos θ *b) mg sin θ* *c) mg sec θ* *d) mg cosec θ*

(Ans. : c)

Q.26 *In Q.24, if P = mg, then the angle θ for equilibrium is _____.*

a) 30° *b) 45°* *c) 60°* *d) 90°* **(Ans. : b)**

Q.27 *The normal reaction on the 100 N block A shown in Fig. 5.8 is _____.*

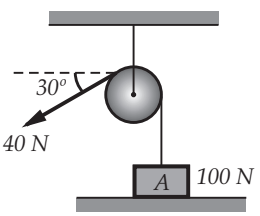


Fig. 5.8

a) 100 N *b) 50 N*

c) 60 N *d) 80 N*

(Ans. : c)

The tensile force in cable is 40 N

$\therefore$ $R_A = 100 - 40 = 60$ **N**

Q.28 *A 50 kg mass is suspended from two spring balances as shown in Fig. 5.9 _____.*

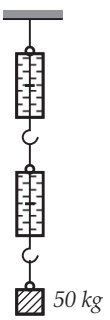


Fig. 5.9

a) both the scales will read 50 kg

b) both the scales will read 25 kg

c) the lower scale will read 50 kg and the upper zero

d) the upper scale will read 50 kg and the lower zero

(Ans. : a)

Q.29 *The horizontal force 'P' required to keep a block of mass 'm' in equilibrium on a frictionless surface inclined at angle 'θ' is _____.*

a) mg tan θ *b) mg cos θ*

c) mg sin θ *d) mg*

(Ans. : a For forces parallel to the inclined plane.

$P \cos\theta - mg \sin\theta = 0$

$\therefore P = mg \tan\theta$)

Q.30 *The tension T_1 in Fig. 5.10 is _____.*

a) mg *b) $\sqrt{3}$ mg*

c) $\frac{mg}{2}$ *d) $\frac{\sqrt{3}}{2}$ mg*

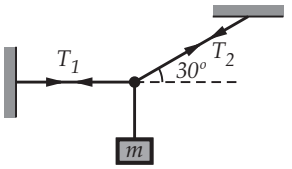


Fig. 5.10

(Ans. : b)

$$T_2 = \frac{mg}{\sin 30} = 2\,mg$$

$$T_1 = T_2 \cos 30 = 2\,mg \times \frac{\sqrt{3}}{2}$$

$$\therefore T_1 = \sqrt{3}\,mg)$$

Q.31 *The two cables AB and BC shown in Fig. 5.11 can withstand a maximum tensile force of 10 kN. The maximum force 'F' that can be applied without breaking the two cables is _____.*

a) 10 kN *b) $10\sqrt{2}$ kN*

c) $20\sqrt{3}$ kN *d) 20 kN*

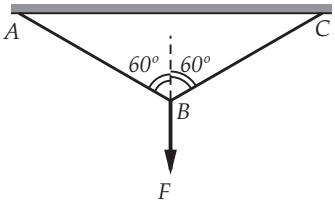


Fig. 5.11

(Ans. : a)

$F = 2 \times 10 \cos 60$

$\therefore F = 10$ kN)

Q.32 *In tug of war, two opposing teams are pulling the rope with equal and opposite forces of 1000 N at each end. The tension in the rope is _____.*

a) 0 *b) 1000 N* *c) 2000 N* *d) 1000 $\sqrt{2}$ N*

(Ans. : b)

Q.33 *Fig. 5.12 shows a cable supporting a block of mass 'm'. Which of the following graphs represents the variation in tension 'T' in the cable with angle 'θ' ?*

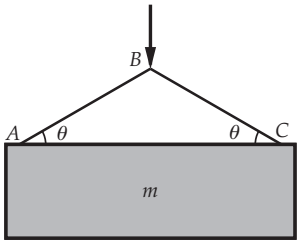


Fig. 5.12

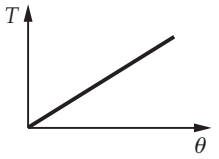


Fig. a

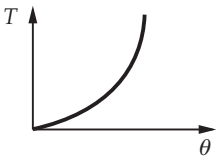


Fig. b

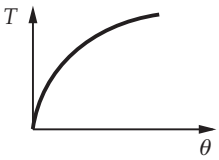


Fig. c

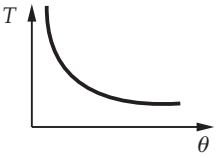


Fig. d

(Ans. : d)

$$2T \sin\theta = mg$$

$$\therefore \quad T = \frac{mg}{2 \sin\theta}$$

For $\theta = 0$, $T = \infty$

For $\theta = 90°$, $T = \frac{mg}{2}$)

Q.34 *Two forces each of magnitude 'P' are applied at a point with angle 'θ' between them. The magnitude of a third force 'F' required for equilibrium is ____.*

a) $2P \cos\frac{\theta}{2}$ *b)* $2P \cos\theta$ *c)* $2P \sin\theta$ *d)* $2P$

(Ans. : a)

Q.35 *In Q.34 the value of angle α made by the third force with the first force is*

a) $0°$ *b)* $90°$ *c)* $180 + \frac{\theta}{2}$ *d)* $180 + \theta$

(Ans. : c)

Q.36 *If three concurrent forces, each of magnitude P, are in equilibrium, the angle made by the three forces with positive x-axis can be _____ .*

a) $30°, 60°, 90°$ *b)* $0°, 90°, 270°$ *c)* $30°, 150°, 270°$ *d)* $0°, 180°, 270°$

(Ans. : c)

Q.37 *A 100 N block is to be held in equilibrium in the position shown in Fig. 5.13. The value of 'θ' for equilibrium is _____.*

a) $30°$ *b)* $45°$

c) $60°$ *d)* $20°$

(Ans. : a)

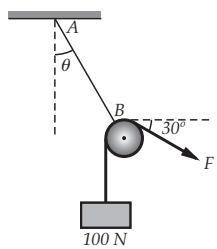


Fig. 5.13

Q.38 *In Q. 37 the magnitude of force 'F' is _____.*

a) 50 N *b)* 75 N

c) 100 N *d)* $100\sqrt{2}$ N

(Ans. : b)

Q.39 *In Q.37, the tension in cable AB is _____.*

a) $100\sqrt{3}$ *N* *b)* $50\sqrt{3}$ *N*

c) 100 N *d)* $100\sqrt{2}$ *N*

(Ans. : a)

Q.40 *The spring shown in Fig. 5.14 has a spring constant of 50 N/m and an undeformed length of 3 m. It is connected to a collar which moves on a frictionless horizontal rod. If the collar is displaced horizontally by 4 m, the force in the spring is _____.*

a) 100 N ← *b) 100 N* →

c) 100 N, 36.87^o *d) 100 N,* 36.87^o

(Ans. : d)

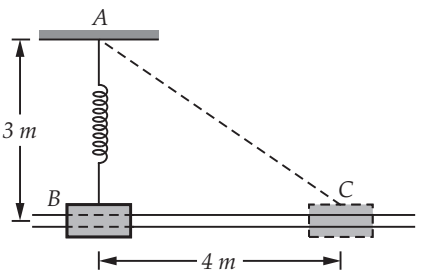


Fig. 5.14

Q.41 *If the maximum tension in cable ABC is 100 kN, the maximum weight 'W' that can be lifted with constant velocity as shown in Fig. 5.15 is _____.*

a) 200 kN *b) 100 kN*

c) $100\sqrt{3}$ *kN* *d)* $200\sqrt{3}$ *kN*

(Ans. : (b) W = 2 × 100 cos 60 = 100 kN)

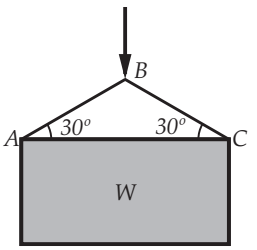


Fig. 5.15

Q.42 *A 500 N roller rests on an inclined plane and is supported by a cable AB as shown in Fig. 5.16. The tension in cable AB is _____.*

a) 500 N *b) 250 N*

c) 750 N *d)* $500\sqrt{3}$ *N*

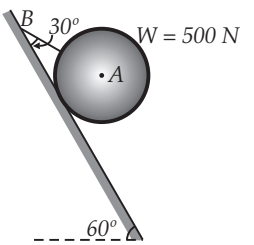


Fig. 5.16

(Ans. : (a)

The forces on the roller are as shown in Fig. 5.17.

∴ R = T = 500 N

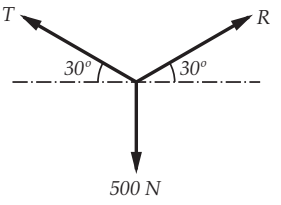


Fig. 5.16 (a)

Q.43 *The reaction on the roller in Q.42 is _____.*

a) 500 N *b) 250 N*

c) 750 N *d)* $500\sqrt{3}$ *N*

(Ans. : a)

Q.44 *Three identical rollers of mass 'm' are stacked as shown in Fig. 5.11. B and C are connected by a cable. The reaction between B and C will be _____.*

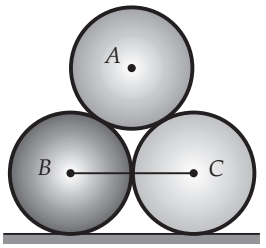


Fig. 5.17

a) mg *b)* $\frac{mg}{\sqrt{3}}$

c) $\frac{\sqrt{3}\,mg}{6}$ *d)* 0

(Ans. : d)

Q.45 *In Q.44, the tension in BC is _____.*

a) mg *b)* $\frac{mg}{\sqrt{3}}$

c) $\frac{\sqrt{3}\,mg}{6}$ *d)* 0

(Ans. : c)

The F.B.D. of A and B are shown in Fig. 5.18. From F.B.D. of A,

$$\sum F_x = 0 \Rightarrow R_{AB} = R_{AC}$$

$$\sum F_y = 0 \Rightarrow 2\,R_{AB} \sin 60 = mg$$

$$\therefore R_{AB} = \frac{mg}{\sqrt{3}}$$

. From F.B.D. of B, $\sum F_x = 0$ **:** $T - R_{AB} \cos 60 = 0$

$$\therefore \quad T = \frac{mg}{2\sqrt{3}} = \frac{\sqrt{3}\,mg}{6}$$

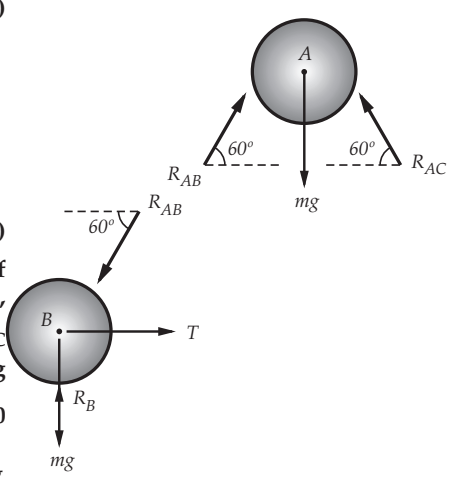


Fig. 5.18

Q.46 *In Q.44, the reaction on B from the horizontal surface is*

a) mg *b)* $3\,\frac{mg}{2}$ *c)* $\frac{mg}{\sqrt{3}}$ *d)* $\frac{\sqrt{3}\,mg}{6}$

(Ans. : b) From F.B.D. of B,

$$\sum F_y = 0$$

$$-R_{AB} \sin 60 + R_B - mg = 0$$

$$\therefore \quad R_B = \frac{mg}{\sqrt{3}} \times \frac{\sqrt{3}}{2} + mg$$

$$\therefore \quad R_B = \frac{3\,mg}{2}$$

Q.47 *A block of weight 'W' is kept on a frictionless inclined plane making angle 'θ' with the horizontal. The horizontal force 'P' required to keep the block in equilibrium is ____.*

a) $W \sin \theta$ *b)* $W \cos \theta$

c) $W \tan \theta$ *d)* $W \cot \theta$

(Ans. : c

From Fig. 5.14

$$\sum F_x = 0$$

$$P \cos \theta - W \sin \theta = 0$$

$$\therefore P = W \tan \theta$$

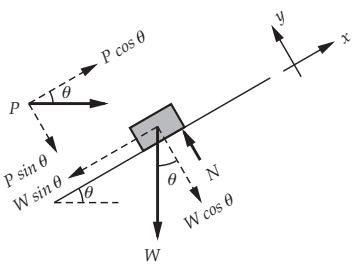


Fig. 5.14

Q.48 *A bar AB of length 'l' and weight 'W' is hinged at A and supported by a horizontal cable at B. If the bar makes angle 'θ' with horizontal, the tension in the cable is _____.*

a) $\frac{W}{2\tan\theta}$ *b)* $\frac{W}{2}\tan\theta$

c) $W \tan\theta$ *d)* $\frac{W}{\tan\theta}$

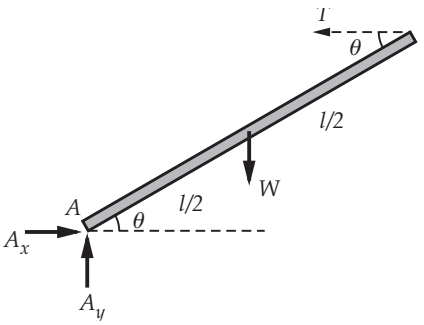


Fig. 5.20

(Ans. : a From Fig. 5.20

$$\sum M_A = 0$$

$$T \times l \sin\theta - W \times \frac{l}{2}\cos\theta = 0$$

$$\therefore \quad T = \frac{W}{2\tan\theta}\textbf{)}$$

Q.49 *The horizontal component of reaction at A in Q.48 is _____.*

a) W *b)* $\frac{W}{2}\tan\theta$ *c)* $\frac{W}{2\tan\theta}$ *d)* $W \tan\theta$

(Ans. : c

$$\sum F_x = 0 :$$

$$A_x - T = 0$$

$$\therefore \quad A_x = \frac{W}{2\tan\theta}\textbf{)}$$

Q.50 *The vertical component of reaction at A in Q.48 is _____.*

a) W *b)* $\frac{W}{2}\tan\theta$ *c)* $\frac{W}{2\tan\theta}$ *d)* $W \tan\theta$

(Ans. : a

$$\sum F_y = 0 :$$

$$A_y - W = 0$$

$$\therefore \quad A_y = W\textbf{)}$$

Q.51 *A cylinder of weight 'W' is supported between a smooth vertical wall and a smooth plane inclined at angle θ with the horizontal. The reaction on the cylinder from the inclined plane is _____.*

a) $W \cos\theta$ *b)* $W \sin\theta$

c) $W \operatorname{cosec}\theta$ *d)* $W \sec\theta$

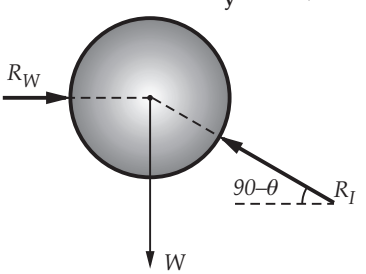


Fig. 5.21

(Ans. : d
From Fig. 5.21

$$\sum F_y = 0 :$$

$$R_I \sin(90-\theta) - W = 0$$

$$\therefore \quad R_I = W \sec\theta\textbf{)}$$

Q.52 *In Q.51, the reaction from the wall is _____.*

a) W cot θ *b) W tan θ* *c) W sec θ* *d) W cosec θ*

(Ans. : b

$$\sum F_x = 0 :$$

$$R_W - R_I \cos(90 - \theta) = 0$$

$$\therefore \quad R_W = W \sec\theta \times \sin\theta$$

$$= W \tan\theta)$$

Q.53 *A cylinder of weight 'W' is kept between two smooth inclined planes, each inclined at angle θ with the horizontal. The reaction from each inclined plane on the cylinder is ____.*

a) $\frac{W}{2\cos\theta}$ *b)* $\frac{W}{2}\cos\theta$ *c)* $\frac{W}{2\sin\theta}$ *d)* $\frac{W}{2}\sin\theta$

(Ans. : a

$$\sum F_x = 0$$

$$\Rightarrow R_1 = R_2$$

$$\sum F_y = 0 :$$

$$2 R_1 \sin(90 - \theta) = W$$

$$\therefore \quad R_1 = \frac{W}{2\cos\theta})$$

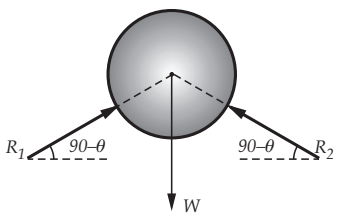


Fig. 5.22

Q.54 *Three forces of magnitudes F_1 = 12 kN, F_2 = 5 kN and F_3 = 13 kN act at on a particle in a plane. The particle will remain in equilibrium only if the angle between F_1 and F_2 is _____.*

a) 30° *b) 45°* *c) 60°* *d) 90°*

(Ans. : d)

Q.55 *Three forces of magnitudes F_1 = 100 N, F_2 = 100 N and F_3 = 100 $\sqrt{3}$ N act on a particle in a plane. The particle will remain in equilibrium only if the angle between F_1 and F_2 is _____.*

a) 30° *b) 45°* *c) 60°* *d) 90°* **VTU : Feb.-11**

(Ans. : c)

Q.56 *Three forces act in the x-y plane and are in equilibrium. Horizontal force $\vec{F}_1$ of magnitude 30 N acts at point (5, 0) a vertical force $\vec{F}_2$ acts at point (0, 7) and a third force $\vec{F}_3$ acts at point ($\sqrt{3}$, 1). $\vec{F}_1$ and $\vec{F}_2$ act along the positive x and y-axes respectively. The direction of $\vec{F}_3$ is _____.*

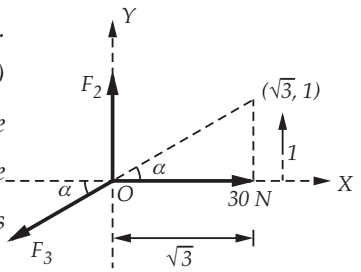


Fig. 5.23

a) 30°, first quadrant *b) 30°, third quadrant*

c) 60°, first quadrant *d) 60°, third quadrant*

(Ans. : b

The forces are concurrent at origin as shown in Fig. 5.23.

$$\therefore \quad \alpha = x$$

$$\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ \quad \swarrow)$$

Q.57 *In Q.56, the magnitude of* $\vec{F}_2$ *is _____.*

a) 26 N *b) 34.64 N* *c) 17.32 N* *d) 13 N*

(Ans. : c

$$\frac{F_2}{\sin 150} = \frac{30}{\sin 120}$$

$$\therefore \quad F_2 = 17.32 \text{ N}$$

Q.58 *In Q.56, the magnitude of* $\vec{F}_3$ *is _____.*

a) 26 N *b) 34.64 N* *c) 17.32 N* *d) 13 N*

(Ans. : b

$$\frac{F_3}{\sin 90} = \frac{30}{\sin 120}$$

$$\therefore \quad F_3 = 34.64 \text{ N)}$$

Q.59 *Which of the following systems of coplanar concurrent forces can remain in equilibrium ?*

a) 20 N, 70 N, 120 N *b) 10 N, 90 N, 200 N*

c) 50 N, 50 N, 150 N *d) 30 N, 50 N, 50 N*

(Ans. : d)

Q.60 *Which of the following system of coplanar concurrent force systems cannot remain in equilibrium ?*

a) 30 N, 30 N, 40 N *b) 30 N, 30 N, 70 N*

d) 30 N, 30 N, 10 N *d) 30 N, 30 N, 30 N*

(Ans. : b)

Q.61 *A horizontal beam AB of length 9 m is subjected to two vertically downward forces of 100 N and 200 N at A and B respectively. The magnitude of the third force, direction and its point of application from A are _____.*

a) 300 N ↑, 6 m *b) 300 N ↑, 3 m* *c) 300 N ↓, 6 m* *d) 300 N ↓, 3 m*

(Ans. : a)

Q.62 *A 1 kg object is falling with a constant speed 10 m/s. The net force on the object is _____.*

a) 9.81 N *b) 10 N* *c) 0.1 N* *d) 0*

(Ans. : d)

Q.63 *Lami's theorem is valid for _____ forces in equilibrium.* **VTU : Dec.13**

a) 2 *b) 3* *c) 4* *d) any number of*

(Ans. : b)

Q.64 *A particle in equilibrium has _____ acceleration.*

a) constant *b) zero* *c) increasing* *d) none of these*

(Ans. : b)

Q.65 *A particle subjected to non-zero and constant resultant force has _____ acceleration.*

a) constant *b) zero* *c) increasing* *d) none of these*

(Ans. : a)

Q.66 *For equilibrium of a collinear force system, the number of equations is _____.*

a) 1 *b) 2* *c) 3* *d) none of these*

(Ans. : a)

Q.67 *The minimum number of coplanar and non-collinear forces required for equilibrium is _____.*

a) 2 *b) 3* *c) 4* *d) 1*

(Ans. : b)

Q.68 *A particle is acted upon by two non-collinear forces* $\vec{F_1}$ *and* $\vec{F_2}$. *The third force* $\vec{F_3}$ *required for equilibrium must have magnitude _____.*

a) same as $|\vec{F_1}|$ *b) same as* $|\vec{F_2}|$

c) same as $|\vec{F_1} + \vec{F_2}|$ *d) same as* $|\vec{F_1} - \vec{F_2}|$

(Ans. : c)

Q.69 *A rigid body is in equilibrium under the action of three forces. It implies that the forces must be such that the _____ .*

a) resultant is zero *b) equilibriant is zero* *c) both a and b* *d) none of these*

VTU : Aug.-10

(Ans. : c)

Q.70 *Lami's theorem _____ .*

a) relates the forces with the sines of angles

b) relates the action of three concurrent forces and sines of angles

c) may be applied to consider relationship between forces

d) may be applied for a body which may or may not be in equilibrium

VTU : Aug.-10

(Ans. : b)

Q.71 *If sum of all the forces acting on a body is zero, it may be concluded that the body ___ .*

a) must be in equilibrium *b) cannot be in equilibrium*

c) may be in equilibrium provided the forces are concurrent

d) may be in equilibrium provided the forces are parallel

VTU : Aug.-10, 11

(Ans. : c)

Q.72 *The force that cancels the effects of the force system acting on the body is known as ___ .*

a) resultant *b) neutral force* *c) balancing force* *d) equilibriant*

VTU : Aug.-11

(Ans. : d)

Q.73 *For a system of coplanar parallel forces to be in equilibrium*

a) the resultant force must vanish alone is sufficient

b) the resultant couple must vanish alone is sufficient

c) both resultant force and the resultant couple must vanish

d) none of the above

VTU : Aug.-11

(Ans. : c)

Q.74 *A block of weight, W, is kept on a frictionless inclined plane making an angle, θ with the horizontal. The horizontal force, P, required to keep the block in equilibrium is _____ .*

VTU : Dec.-11, Marks 2

A) W sin θ *B) (W/2) tan θ* *C) W tan θ* *D) (W/tan θ)*

(Ans. : c)

Q.75 *A particle is acted upon by two non-collinear forces $\vec{F}_1$ and $\vec{F}_2$. The third force $\vec{F}_3$ required for equilibrium must be applied _____.*

a) in the direction of $\vec{F}_1$ *b) in the direction of $\vec{F}_2$*

c) in the direction of $\vec{F}_1 + \vec{F}_2$ *d) opposite to direction of $\vec{F}_1 + \vec{F}_2$*

(Ans. : d)

Q.76 *If three coplanar concurrent forces are in equilibrium, the magnitude of any force is proportional to _____.*

a) magnitudes of other forces *b) magnitude of resultant force*

c) angle between the other two forces *d) sine of angle between the other two forces*

(Ans. : d)

Q.77 *Forces in cables are always _____.*

a) tensile *b) compressive* *c) zero* *d) none of these*

(Ans. : a)

Q.78 *If three coplanar concurrent forces are in equilibrium every force is _____ of the other two forces.*

a) resultant *b) equilibrant*

c) equal to magnitude *d) none of the above*

(Ans. : b)

Q.79 *If forces F_1 and F_2 act along a straight line and F_3 is inclined at angle θ with F_1, then for equilibrium _____.*

a) $F_3 = 0$ *b) $F_3 = F_1 \cos\theta$* *c) $F_3 = F_1 \sin\theta$* *d) $F_3 = F_2 \sin\theta$*

(Ans. : a)

Q.80 *A man holds a spring balance from which a 10 kg weight is suspended. The force in the spring will be _____ .*

a) 10 N *b) 196.2 N* *c) 98.1 N* *d) 0*

(Ans. : c)

Q.81 *Force in a straight rod can be _____ .*

a) tensile *b) compressive* *c) non-axial* *d) Any of the above*

(Ans. : d)

Q.82 *For an object kept on a inclined plane, its weight acts _____ .*

a) perpendicular to the plane *b) parallel to the plane*

c) vertical *d) horizontal*

(Ans. : c)

Q.83 *Free body diagram can be drawn for _____ .*

a) a system of bodies *b) a isolated body*

c) a part of the body *d) any of the above*

(Ans. : d)

Q.84 *For equilibrium of a body subjected to coplanar non concurrent forces, the _____ .*

VTU : Dec.-11

a) $\sum Fx = 0$ *and* $\sum Fy = 0$ *b)* $\sum Fx = 0$ *and* $\sum M = 0$

c) $\sum M = 0$ *d)* $\sum Fx = 0$ *and* $\sum Fy = 0$ $\sum M = 0$.

(Ans. : d)

Q.85 *Conditions of equilibrium for a coplanar concurrent force system is _____.*

VTU : Dec.-13

a) one *b) two* *c) three* *d) four*

(Ans. : b)

Q.86 *If three forces are acting at a point are in equilibrium, out of which two are acting in the same line, then the third force is _____.* **VTU : Dec.-13**

a) maximum *b) minimum* *c) zero* *d) none of these*

(Ans. : c)

Q.87 *A rigid body is in equilibrium if the resultant force of concurrent force system is _____.*

VTU : Dec.-13

a) positive *b) negative* *c) zero* *d) none ofthese*

(Ans. : c)

Q.88 *The necessary condition of equilibrium of a coplanar concurrent force system is algebric sum of _____ must be zero.*

A) Horizontal and verical forces *B) Moment of forces*

C) Horizontal, vertical and moment of forces D) None of these **VTU : June-13**

(Ans. : a)

❑❑❑

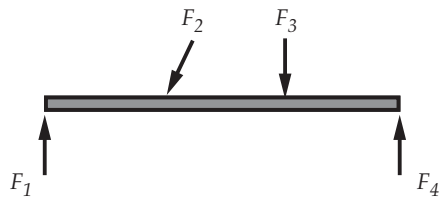
Notes

6 Beams

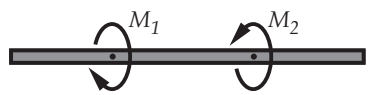
Chapter at a Glance

- For a simple support, the support reaction is perpendicular to the supporting surface.
- In a roller reaction, the reaction is perpendicular to the surface on which the roller is resting and is directed towards the beam.
- At a hinged support, the reaction is represented by two mutually perpendicular x and y components.
- At a fixed support, there are two mutually perpendicular x and y components of reaction and a reaction moment.
- **Loads on beams :**

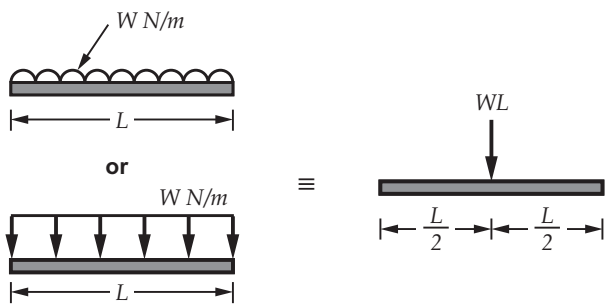
i) Point loads



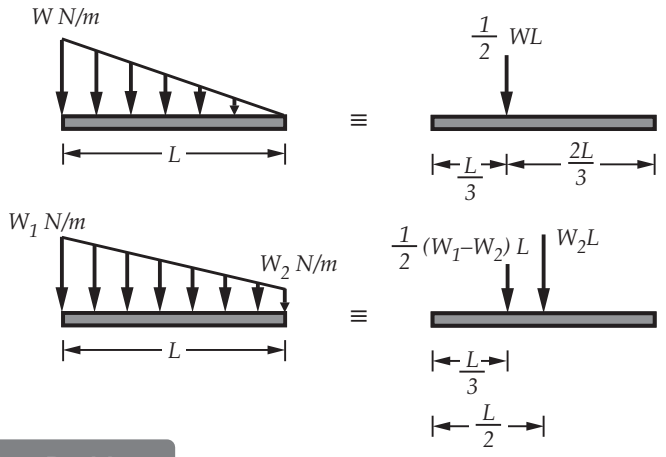
ii) Couple moments



iii) Uniformly distributed load (u.d.l)



iv) Uniformly varying load (u.v.l)



Method for Solving Problems

1. Draw free body diagram of the beam. Reduce all distributed loads to point loads.
2. Use equations of static equilibrium.

$$\sum F_x = 0$$
$$\sum F_y = 0 \quad \text{and}$$
$$\sum M = 0$$

to solve for the unknowns.

3. For compound beams draw separate F.B.D. of each beam showing equal and opposite reactions at common supports. Use equations of static equilibrium for each F.B.D. to solve for unknowns. Always start with that F.B.D. which has least number of unknowns.

Important Theory Questions and Answers

Q.1 *Explain different types of supports and reactions.* **VTU : Aug.-08, Feb.-09, June-13**

Ans. : The different types of supports and reactions are as follows :

1) Simple support : The beam rests on a rigid surface as shown in Fig. 6.1 (a). The support reaction is perpendicular to the supporting surface towards the beam as shown in Fig. 6.1 (b).

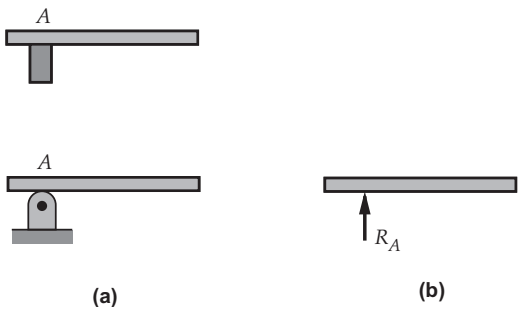


Fig. 6.1

2) **Roller support :** When a beam is supported on a roller kept on a rigid surface, the beam can roll on the roller parallel to the surface on which the roller rests. The reaction is perpendicular to the surface on which the roller is resting and is directed towards the beam. The symbols for roller support are shown in Fig. 6.2 (a) and the corresponding reactions in Fig. 6.2.(b).

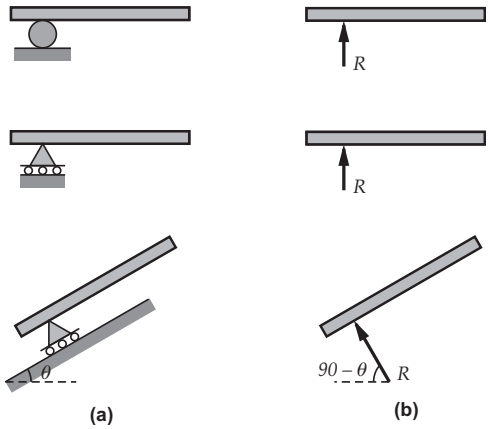


Fig. 6.2

3) **Hinged (or pinned) support :** The hinged support allows the beam to rotate about the hinge but does not allow translation in the plane. The reaction is represented by two mutually perpendicular x and y components. The symbols for the hinged support in beams are shown in Fig. 6.3 (a) and the corresponding reaction in Fig. 6.2.3 (b).

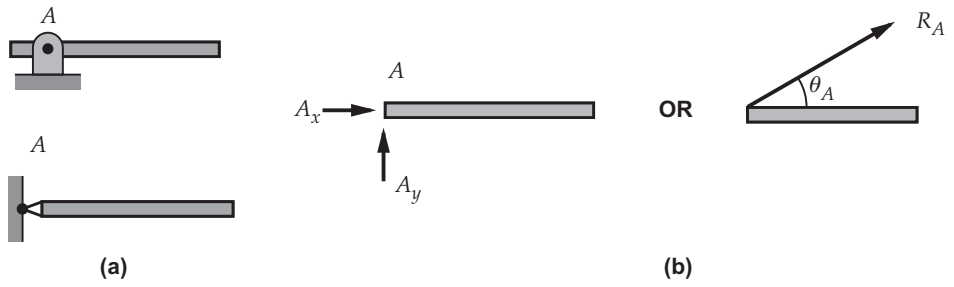


Fig. 6.3

4) **Fixed support :** This type of support does not allow translation in plane and rotation about the support. Hence the reaction is a combination of a force which can be directed any where in the plane depending upon applied forces and a reaction moment. The reaction force can be replaced by two mutually perpendicular components. The symbol for the fixed support is shown in Fig. 6.4 (a) and the corresponding reactions in Fig. 6.4 (b).

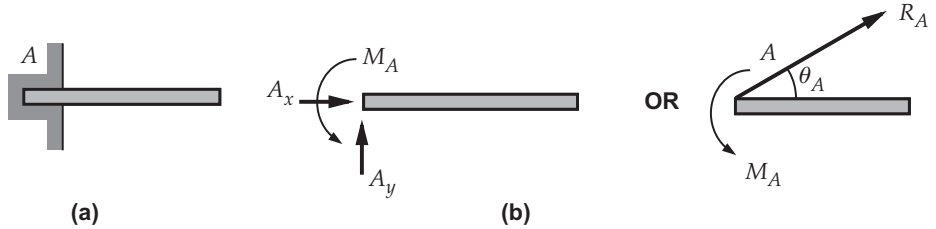


Fig. 6.4

Important Solved Examples

Example 6.1 *For the beam with the loading shown in Fig. 6.5, determine the reactions at the supports.*

VTU : Feb.-04, Dec.-11, Jan.-13, Marks 10

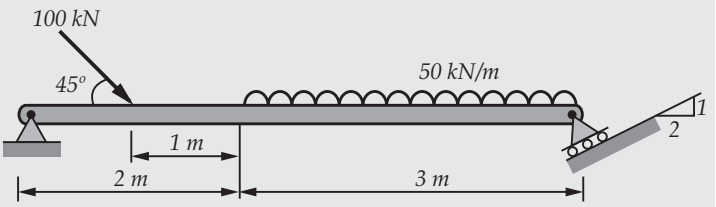


Fig. 6.5

Solution : The F.B.D. of beam is shown in Fig. 6.5 (a). The angle made by roller reaction with horizontal is $\tan^{-1}\left(\frac{2}{1}\right) = 63.43°$.

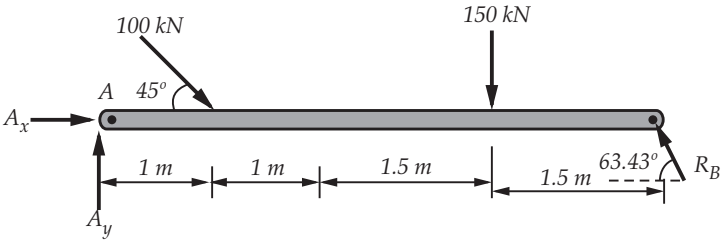


Fig. 6.5 (a)

$$\sum M_A = 0 :$$

$$-(100 \sin 45)(1) - (150)(3.5) + (R_B \sin 63.43)(5) = 0$$

$$\therefore \quad R_B = 133.21 \text{ kN}, 63.43^\circ \searrow$$

$$\sum F_x = 0 :$$

$$A_x + 100 \cos 45 - R_B \cos 63.43 = 0$$

$$\therefore \quad A_x = -11.127 \text{ kN}$$

$$\sum F_y = 0 :$$

$$A_y - 100 \sin 45 - 150 + R_B \sin 63.43 = 01$$

$$A_y = 101.57 \text{ kN}$$

$$R_A = \sqrt{A_x^2 + A_y^2}$$

$$= \sqrt{11.127^2 + 101.57^2}$$

$$\therefore \quad R_A = 102.18 \text{ kN}$$

$$\theta_A = \tan^{-1}\left(\frac{|A_y|}{|A_x|}\right)$$

$$= \tan^{-1}\left(\frac{101.57}{11.127}\right)$$

$$\therefore \quad \theta_A = 83.75^\circ \searrow$$

Example 6.2 *Determine the reactions at the supports for the system shown in Fig. 6.6.*

VTU : Aug.-10, Marks 12

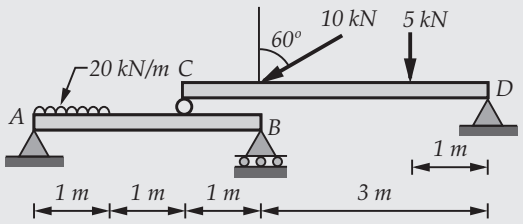


Fig. 6.6

Solution : The free body diagrams of beams *AB* and *CD* are shown in Fig. 6.6 (a).

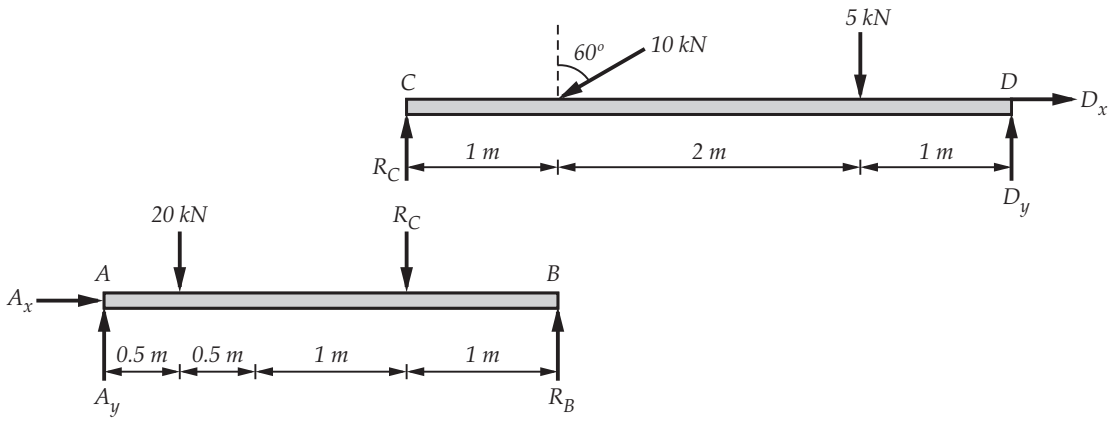


Fig. 6.6 (a)

For *CD*,

$\sum M_D = 0$:

$-(R_C)(4)+(10\cos 60)(3)+(5)(1) = 0$

$\therefore$ $\boxed{R_C = 5 \text{ kN}}$

$\sum F_x = 0$:

$D_x - 10\sin 60 = 0$

$\therefore$ $D_x = 8.66$ kN

$\sum F_y = 0$:

$R_C - 10\cos 60 - 5 + D_y = 0$

$\therefore$ $D_y = 5$ kN

$R_D = \sqrt{D_x^2 + D_y^2} = \sqrt{8.66^2 + 5^2}$

$\therefore$ $\boxed{R_D = 10 \text{ kN}}$

$$\theta_D = \tan^{-1}\left(\frac{|D_y|}{|D_x|}\right) = \tan^{-1}\left(\frac{5}{8.66}\right)$$

$\therefore$ $\theta_D = 30^\circ$

For *AB*,

$$\sum M_A = 0 :$$

$$-(20)(0.5)-(R_C)(2)+(R_B)(3) = 0$$

$\therefore$ $R_B = 6.67$ kN ↑

$$\sum F_x = 0 :$$

$$A_x = 0$$

$$\sum F_y = 0 :$$

$$A_y - 20 - R_C + R_B = 0$$

$\therefore$ $A_y = 18.33$ kN

$\therefore$ $R_A = 18.33$ kN ↑

Example 6.3 *The cantilever beam shown in Fig. 6.7 is fixed at 'A' and is free at 'B'. Determine the reaction when it is loaded as shown.* **VTU : Dec.-13, Marks 6**

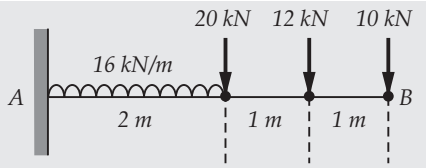


Fig. 6.7

Solution : The F.B.D. of beam is shown in Fig. 6.7 (a).

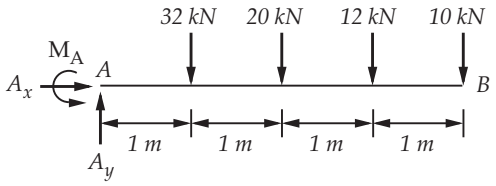


Fig. 6.7(a)

$$\sum F_x = 0 : \quad A_x = 0$$

$$\sum F_y = 0 :$$

$$A_y - 32 - 20 - 12 - 10 = 0$$

$\therefore$ $A_y = 74$ KN

$\therefore$ $R_A = 74$ KN ↑

$\therefore$ $\sum M_A = 0 :$

$$M_A - (32)(1) - (20)(2) - (12)(3) - (10)(4) = 0$$

$\therefore$ $M_A = 148$ kNm

Important Multiple Choice Questions

Q.1 *Statically determinate beams are _____.*

a) the beams which can be analyzed completely using equations of equilibrium

b) the beams which can be analyzed using static equations

c) fixed beams *d) none of the above*

VTU : Feb.-09, June-13

(Ans. : a)

Q.2 *Fixed beams are _____.*

a) one end is fixed and the other is simply supported

b) both ends are fixed *c) both ends are roller support*

d) one end is fixed and the other is free

VTU : Feb.-09, 11, June-13

(Ans. : b)

Q.3 *The number of reaction components at fixed end of a beam are _____.*

a) 2 *b) 3* *c) 4* *d) none of the above*

VTU : Feb.-09, Dec.-11,13, June-13

(Ans. : b)

Q.4 *u.d.l. stands for _____.*

a) uniform dead load *b) uniformly distributed load*

c) uniform door load *d) all the above*

VTU : Jan.-09,10, June-13, Dec.-13

(Ans. : b)

Q.5 *A reaction line at roller support with respect to plane of contact is _____.*

a) oblique *b) obtuse* *c) perpendicular* *d) none*

VTU : Aug.-09, Jan.-13

(Ans. : c)

Q.6 *Support reactions for statically determinate beams can be determined by applying _____.*

a) conditions of static equilibrium *b) Lamis theorem*

c) Varignon principle *c) none of the above*

VTU : Aug.-09

(Ans. : a)

Q.7 *When load acts at constant rate over given length of beam it is called _____*

a) point load. *b) u.d.l.* *c) u.v.l.* *d) none.*

VTU : Aug.-09, Jan.-13

(Ans. : b)

Q.8 *A beam having one end hinged support and other roller support subjected to vertical loading can be regarded as _____.*

a) fixed beam *b) cantilever beam*

c) simply supported beam *d) none of these* **VTU : Aug.-09**

(Ans. : c)

Q.9 *The number of reaction components at a hinged end of a beam are _____.*

a) 0 *b) 2* *c) 3* *d) 1*

VTU : Feb.-10, June-12, Dec.-13

(Ans. : b)

Q.10 *A cantilever beam is one in which _____.*

a) both ends are fixed *b) both ends are hinged*

c) one end is fixed and the other is free

d) one end is fixed and the other is simply supported

VTU : Feb.-10, Dec.-11, June-12

(Ans. : c)

Q.11 *At the fixed end of cantilever, number of unknown reaction components are _____.*

a) 1 *b) 2* *c) 3* *d) 4*

VTU : Jan.-10,13

(Ans. : c)

Q.12 *If one end of a beam is fixed and the other is supported by a roller, it is known as _____.*

a) cantilever beam *b) fixed beam*

c) propped cantilever beam *d) overhanging beam*

(Ans. : c)

Q.13 *A beam with three or more than three supports is known as _____ .*

a) cantilever beam *b) fixed beam* *c) propped beam* *d) continuous beam*

(Ans. : d)

Q.14 *The number of equations available for solving a beam supported by rollers at both ends is _____.*

a) 1 *b) 2* *c) 3* *d) 4*

(Ans. : b)

Q.15 *A simply supported beam cannot remain in equilibrium if only a _____.*

a) inclined force is applied *b) vertically upward force is applied*

c) moment is applied *d) any of the above*

(Ans. : d)

Q.16 *A simply supported beam AB of length 5 m is acted upon by a point load of 10 kN at a distance of 2 m from A. The reactions at A and B respectively are _____.*

VTU : Dec.-11

a) 4 kN, 6 kN *b) 6 kN, 4 kN* *c) 5 kN, 5 kN* *d) 10 kN, 0*

(Ans. : b

For Fig. 6.8.

$$\sum M_A = 0 :$$

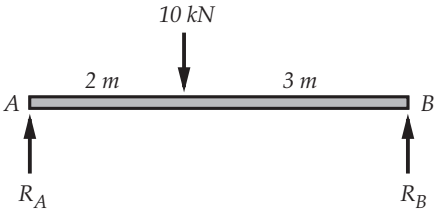
$$R_B \times 5 - 10 \times 2 = 0$$

$$\therefore \quad R_B = 4 \text{ kN}$$

$$\sum F_y = 0 :$$

$$R_A + R_B - 10 = 0$$

$$\therefore \quad R_A = 6 \text{ kN)}$$



Q.17 *A simply supported beam AB of length 5 m is acted upon by a uniformly distributed load of 2 kN/m. The reactions at A and B respectively are _____.*

a) 4 kN, 6 kN *b) 6 kN, 4 kN,* *c) 5 kN, 5 kN* *d) 1 kN, 1 kN*

(Ans. : c

From Fig. 6.9.

$R_A = R_B = 5$ kN)

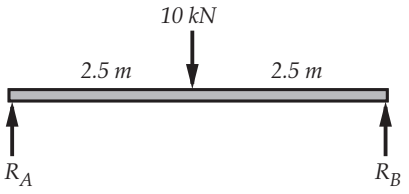


Fig. 6.9

Q.18 *A simply supported beam AB of length 6 m is acted upon by a uniformly varying load with intensity 0 at A and 10 kN/m at B. The reactions at A and B respectively are ____.*

a) 20 kN, 10 kN *b) 10 kN, 20 kN* *c) 15 kN, 15 kN* *d) 20 kN, 20 kN*

(Ans. : b For Fig. 6.10.

$$\sum M_B = 0 :$$

$$30 \times 2 - R_A \times 6 = 0$$

$$\therefore \quad R_A = 10 \text{ kN}$$

$$\sum F_y = 0 \Rightarrow R_B = 20 \text{ kN)}$$

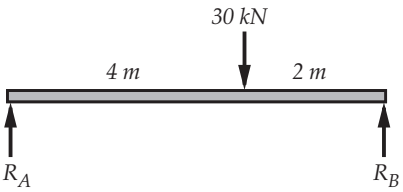


Fig. 6.10

Q.19 *A simply supported beam AB of length 6 m is acted upon by a uniformly varying load with intensity of 10 kN/m of A and 20 kN/m at B. The reactions at A and B respectively are _____.*

a) 40 kN, 50 kN *b) 50 kN, 40 kN* *c) 50 kN, 70 kN* *d) 70 kN, 50 kN*

(Ans. : a) Refer F.B.D. shown in Fig. 6.11)

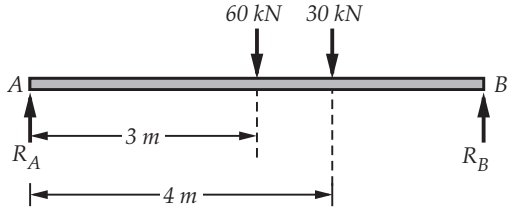


Fig. 6.11

Q.20 *A beam of length 10 m and weight 50 kN is kept on a horizontal surface which offers a uniformly distributed reaction in the vertical direction. The intensity of the vertical reaction is _____.* **VTU : Aug.-10**

a) 50 kN/m b) 25 kN/m c) 10 kN/m d) 5 kN/m

(Ans. : d Refer F.B.D. shown in Fig. 6.12.

W kN/m is the intensity of uniformly

$\therefore \quad W \times 10 = 50$

$\therefore \quad W = 5\ kN/m$

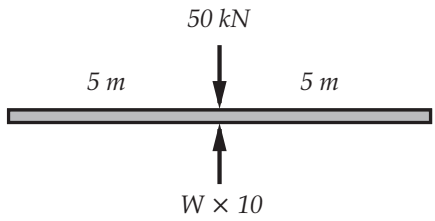


Fig. 6.12

Q.21 *A beam AB of length 5 m is subjected to a clockwise couple moment of 100 Nm at its centre. The end A is hinged and B is supported by a roller. The magnitudes of reaction at A and B are _____.*

a) 50 N, 50 N b) 20 N, 20 N c) 100 N, 100 N d) 75 N, 75 N

(Ans. : b From Fig. 6.13.

$$\sum M_A = 0:$$

$$-100 + R_B \times 5 = 0$$

$$R_B = 20\ N$$

$$\sum F_y = 0 \Rightarrow A_y + R_B = 0$$

$$\therefore \quad A_y = -20\ N$$

$$\sum F_x = 0 \Rightarrow A_x = 0)$$

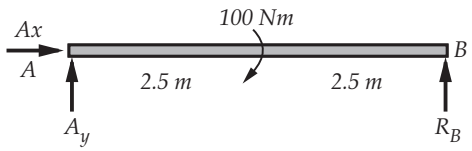


Fig. 6.13

Q.22 *Support reactions for statically indeterminate beams can be determined by applying _____.*

a) conditions of static equilibrium b) Lami's theorem

c) Varignon's theorem d) none of the above

(Ans. : d)

Q.23 *When rate of loading increases or decreases at a constant rate over given length of beam it is called _____*

a) point load. b) u.d.l. c) u.v.l. d) none of the above

(Ans. : c)

Q.24 *A beam extending beyond its supports is known as _____ beam.*

a) propped b) continuous c) overhanging d) cantilever

(Ans. : c)

Q.25 *The number of equations for equilibrium of a cantilever beam subjected to only vertical forces and couple moments is _____.*

a) 1 b) 2 c) 3 d) 4

(Ans. : b)

Q.26 *A beam is said to be determinate, if _____ .* **VTU : Aug.-10**

a) the reactions can be determined using the equations of equilibrium

b) the reactions cannot be determined using the equations of equilibrium

c) the reactions can be determined by using the force equilibrium conditions

d) the reactions can be determined by using the moment equilibrium condition only

(Ans. : a)

Q.27 *The beam is neither permitted to move in any direction nor allowed to rotate in the case of _____ .* **VTU : Aug.-10, 11**

a) hinged support b) fixed support c) roller support d) simple support

(Ans. : b)

Q.28 *u.v.l. stands for _____ .* **VTU : Aug.-10**

a) uniform vertical load *b) uniform velocity load*

c) uniform vague load *d) uniformly varying load*

(Ans. : d)

Q.29 *A thin rigid beam hinged at one end and roller-supported at its mid-point is said to be ____ .*

a) a simply supported beam *b) a overhanging beam*

c) a cantilever beam *d) a fixed beam* **VTU : Aug.-10**

(Ans. : b)

Q.30 *GVL stands for _____ .*

a) General Varying Load *b) Gradually Vertical Load*

c) Gradually Varying Load *d) General Variable Load*

VTU : Feb.-11

(Ans. : d)

Q.31 *A cantilever beam of length L has a point load P at the free end. The vertical component of reaction at the fixed end is _____.*

a) P↑ *b) P↓* *c) 0* *d) PL*

(Ans. : a)

Q.32 *A cantilever beam of length L has a point load P at the free end. The horizontal component of reaction at the fixed end is _____.*

a) P *b) –P* *c) 0* *d) PL*

(Ans. : c)

Q.33 *A cantilever beam of length L has a point load P at the free end. The reaction moment at the fixed end is _____.*

a) P *b) 2P* *c) 0* *d) PL*

(Ans. : d)

Q.34 *A cantilever beam of length L has a uniformly distributed load of w per unit run. The vertical component of reaction at the fixed end is _____.*

a) $w\uparrow$ *b) $wL\uparrow$* *c) 0* *d) $wL\downarrow$*

(Ans. : b)

Q.35 *A cantilever beam of length L has a uniformly distributed load of w per unit run. The reaction moment at the fixed end is _____.*

a) wL *b) wL/2* *c) $\frac{wL^2}{2}$* *d) 2wL*

(Ans. : c)

Q.36 *A cantilever beam of length L has a uniformly varying load of w per unit run at free end and zero at the fixed end. The vertical component of reaction at the fixed end is _____.*

a) wL/3 *b) wL* *c) 0* *d) wL/2*

(Ans. : d)

Q.37 *A cantilever beam of length L has a uniformly varying load of w per unit run at free end and zero at the fixed end. The reaction moment at the fixed end is _____.*

a) $\frac{wL^2}{3}$ *b) $\frac{wL^2}{6}$* *c) $\frac{wL^2}{2}$* *d) 2wL*

(Ans. : a)

Q.38 *A cantilever beam of length L has a uniformly varying load of w per unit run at fixed end and zero at the free end. The vertical component of reaction at the fixed end is _____.*

a) wL/3 *b) wL* *c) 0* *d) wL/2*

(Ans. : d)

Q.39 *A cantilever beam of length L has a uniformly varying load of w per unit run at fixed end and zero at the free end. The reaction moment at the fixed end is _____.*

a) $\frac{wL^2}{3}$ *b) $\frac{wL^2}{6}$* *c) $\frac{wL^2}{2}$* *d) 2wL*

(Ans. : b)

Q.40 *For a beam, if one end is supported on roller and the other on hinge, the beam is said to be _____ .* **VTU : Dec.-11**

a) fixed *b) hinged* *c) cantilever* *d) simply supported*

(Ans. : d)

Q.41 *A determinate beam can have_____ number of unknowns.*

a) 2 *b) 3* *c) 1* *d) 4* **VTU : Dec.-13**

(Ans. : b)

❑❑❑

7 Friction

Chapter at a Glance

- The limiting static friction force, which is the maximum value of friction force, is given by

$$(F_r)_{max} = \mu_s N$$

where μ_s is the coefficient of static friction.

- The force of kinetic friction is given by

$$F_k = \mu_k N$$

where μ_k is the coefficient of kinetic friction.

- If ϕ_k is angle of kinetic friction,

$$\mu_k = \tan \phi_k$$

- The angle of repose is equal to the angle of friction.

Method for Solving Problems to Find Whether Object Slides or Not

1) Draw F.B.D. of object without frictional force.
2) Choose x-axis parallel to the direction in which object can move and y-axis perpendicular to it.
3) Use $\sum F_y = 0$ to find normal reaction N.
4) Find limiting static friction force $(F_r)_{max} = \mu_s N$
5) Find $\sum F_x$, excluding frictional force. This will be net force trying to move the object.
6) If $\left|\sum F_x\right| \leq (F_r)_{max}$, object does not slide. In such a case, the actual value of frictional force will have same magnitude as $\sum F_x$ but opposite direction.

   i.e. $F_r = \sum F_x$ in opposite direction.
7) If $\left|\sum F_x\right| > (F_r)_{max}$, object slides and $F_r = \mu_k N$ in a direction opposite to $\sum F_x$.

Method for Solving Problems to Find Whether Object Slides or Overturns

1) Find the value of force P required to overturn the object by taking moment about the right edge B of the object, i.e, use

$$\sum M_B = 0$$

2) Find the value of P required for sliding using $\sum F_x = 0$ and $\sum F_y = 0$.

3) Motion will start for the smaller value of P.

Method for Solving Problems to Find Quantities for Impending Motion

1) Draw FBD of objects.

2) Use $F_r = \mu_s N$

3) Use conditions for equilibrium : $\sum F_x = 0$, $\sum F_y = 0$, $\sum M = 0$

Important Theory Questions and Answers

Q.1 *Explain the terms : i) Angle of repose ii) Cone of friction* **VTU : Aug.-07, Jan-13**

Ans. : i) Angle of repose : If an object is kept on an inclined plane and the angle of inclination θ is increased, motion impends for a certain value of θ known as **angle of repose**. The angle of repose is equal to the angle of friction.

ii) Cone of friction : If direction of applied force P is changed, keeping its angle with the surface of contact same throughout, the resultant R will take different positions in space but making the same angle ϕ with the normal reaction N.

In such a case, R lies on the surface of a cone known as cone of friction as shown in Fig. 7.1.

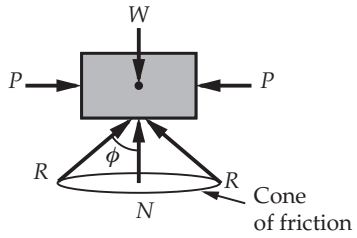


Fig. 7.1

Q.2 *Mention the laws of static friction.* **VTU : Aug.-08, Feb.-09, June-12, 13, Marks 4**

Ans. : The laws of static friction are as follows :

1. Under static conditions, the friction force opposes tendency for relative motion between the two surfaces in contact and acts tangential to the surfaces.
2. The limiting static friction force, which is the maximum value of friction force, is directly proportional to the normal reaction between the two surfaces in contact i.e.

$$(F_r)_{max} \propto N$$

$$\therefore \quad (F_r)_{max} = \mu_s N$$

where μ_s is the coefficient of static friction.

3. Limiting force of static friction is independent of the area of the two surfaces in contact.
4. Limiting force of static friction depends on the nature and material of the two surfaces in contact.

Important Solved Examples

Example 7.1 *A ladder of 4 m weighing 200 N is supported by a horizontal floor and vertical wall shown in Fig. 7.2. If a man of weight 650 N climbs to the top of the ladder, determine the inclination of the ladder with reference to the floor at which the ladder is to be placed to prevent slipping.*

Take $\mu = 0.25$ *for all contact surfaces.*

VTU : Aug.-04, Dec.-11, Jan.-13, Marks 14

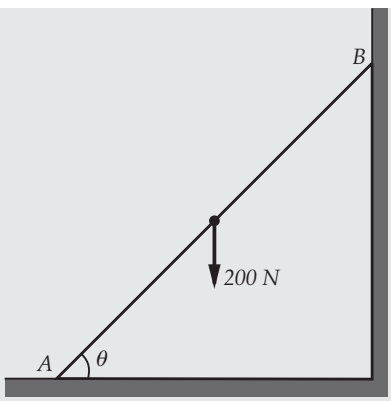


Fig. 7.2

Solution : The F.B.D. the ladder is shown in Fig. 7.2 (a).

$\sum F_x = 0$:

$$0.25\,N_A - N_B = 0 \qquad \ldots (1)$$

$\sum F_y = 0$:

$$N_A + 0.25\,N_B - 200 - 650 = 0$$

$$\therefore \quad N_A + 0.25\,N_B = 850 \qquad \ldots (2)$$

From equation (1) and equation (2),

$$N_B = 200\ N$$

$\sum M_A = 0$:

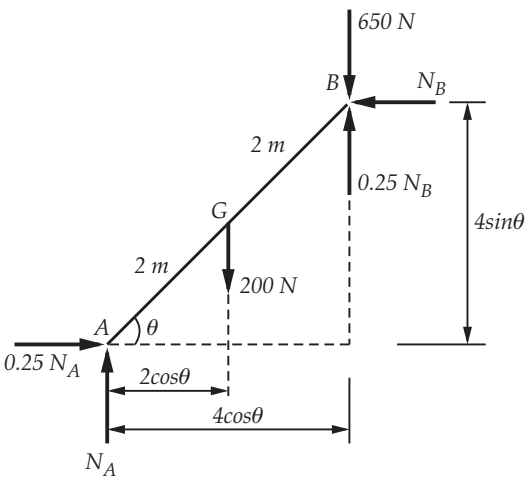


Fig. 7.2 (a)

$$-(200)(2\cos\theta) - (650)(4\cos\theta) + (N_B)(4\sin\theta) + (0.25N_B)(4\cos\theta) = 0$$

$$-400\cos\theta - 2600\cos\theta + 200 \times 4\sin\theta + 200\cos\theta = 0$$

$$800\sin\theta = 2800\cos\theta$$

$$\tan\theta = \frac{2800}{800}$$

$$\therefore \quad \theta = 74.05^{o}$$

Example 7.2 *A small block of weight 1000 N is placed on a 30° incline with a coefficienct of friction at 0.25 as shown in Fig. 7.3 . Determine the horizontal force to be applied for,*

i) The impending motion down the plane and ii) The impending motion up the plane.

VTU : March-05, Feb.-10, Marks 10

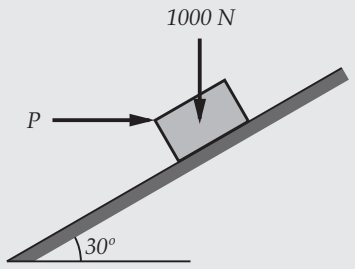


Fig. 7.3

Solution : The F.B.D. of block for the case of impending motion down the plane is shown in Fig. 7.3 (a).

$$\sum F_y = 0 :$$

$$N_1 - P \sin 30 - 1000 \cos 30 = 0$$

$$\therefore \quad N_1 = P \sin 30 + 1000 \cos 30$$

$$\sum F_x = 0 :$$

$$P \cos 30 - 1000 \sin 30 + 0.25\, N_1 = 0$$

$$\therefore$$

$$P \cos 30 - 1000 \sin 30 + 0.25\,(P \sin 30 + 1000 \cos 30) = 0$$

$$P[\cos 30 + 0.25 \sin 30] = 1000 \sin 30 - 0.25 \times 1000 \cos 30$$

$$\therefore \quad \boxed{P = 286.06 \text{ N}}$$

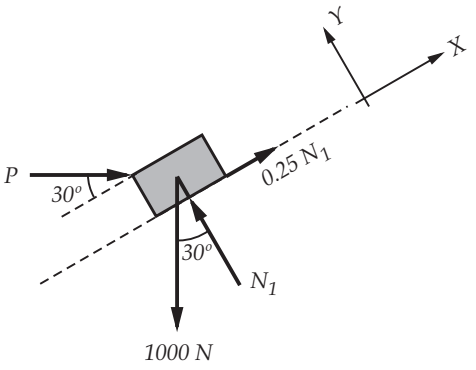


Fig. 7.3 (a)

The F.B.D. of block for the case of impending motion up the plane is shown in Fig. 7.3 (b).

$$\sum F_y = 0 :$$

$$N_1 - 1000 \cos 30 - P \sin 30 = 0$$

$$\therefore N_1 = P \sin 30 + 1000 \cos 30$$

$$\sum F_x = 0 :$$

$$P \cos 30 - 1000 \sin 30 - 0.25\, N_1 = 0$$

$$\therefore P \cos 30 - 1000 \sin 30$$

$$- 0.25\,(P \sin 30 + 1000 \cos 30) = 0$$

$$P(\cos 30 - 0.25 \sin 30) = 1000 \sin 30 + 0.25 \times 1000 \cos 30$$

$$\therefore \quad \boxed{P = 966.91 \text{ N}}$$

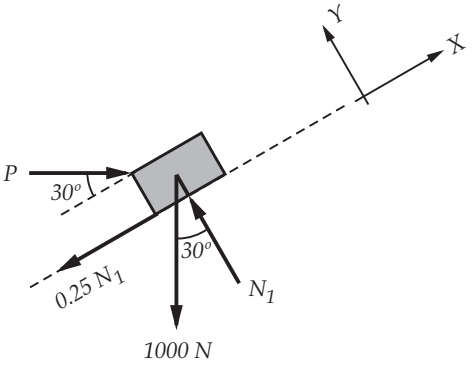


Fig. 7.3 (b)

Example 7.3 *Determine the force P required to start the movement of the wedge as shown in Fig. 7.4. The angle of friction for all surfaces of contact is 15°.* **VTU : Feb.-07, Marks 12**

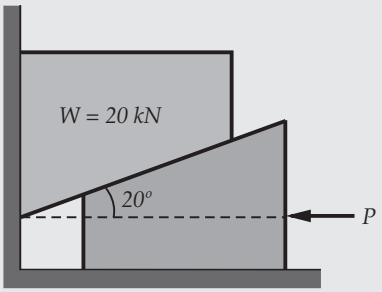


Fig. 7.4

Solution : As $\mu = \tan\phi$ and $\phi = 15^o$,

$$\mu = \tan 15$$

$$\therefore \quad \mu = 0.268$$

As the wedge is pushed towards left, the *20 kN* block will have tendency to move upwards. The free body diagrams are shown in Fig. 7.4 (a).

For F.B.D. of *20 kN* block,

$\sum F_x = 0$:

$$N_1 - N_2 \cos 70 - 0.268\, N_2 \cos 20 = 0 \quad \ldots (1)$$

$\sum F_y = 0$:

$$-0.268\, N_1 + N_2 \sin 70 - 0.268\, N_2 \sin 20 - 20 = 0 \quad \ldots (2)$$

From equation (1) and equation (2)

$$N_2 = 29.033 \text{ kN}$$

For wedge,

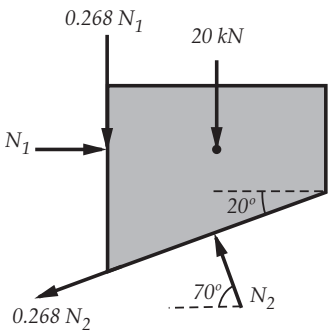
$\sum F_y = 0$:

$$N_3 - N_2 \sin 70 + 0.268\, N_2 \sin 20 = 0$$

$$\therefore \quad N_3 = 24.62 \text{ kN}$$

$$\sum F_x = 0 : -P + N_2 \cos 70 + 0.268\, N_2 \cos 20 + 0.268\, N_3 = 0$$

$$\therefore \quad \boxed{P = 23.84 \text{ kN} \leftarrow}$$



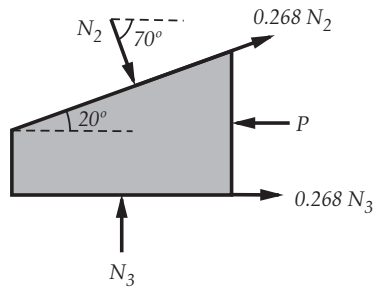


Fig. 7.4 (a)

Example 7.4 *What is the value of 'P' in the system shown in Fig. 7.5 to cause the motion to impend ? Assume the pulley is smooth and the coefficient of friction between the other contact surface is 0.2.*

VTU : Aug.-07, Dec.-13, Marks 14

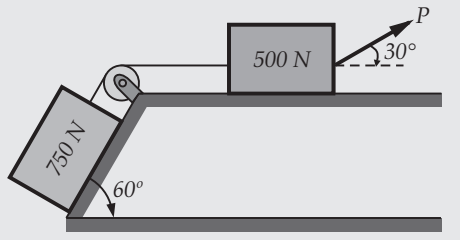


Fig. 7.5

Solution : The free body diagrams of the two blocks are shown in Fig. 7.5 (a).

For F.B.D. of block B,

$\sum F_y = 0$:

$$N_B - 750 \cos 60 = 0$$

$$\therefore \quad N_B = 375 \text{ N}$$

$\sum F_x = 0$:

$$T - 0.2\, N_B - 750 \sin 60 = 0$$

$$\therefore \quad T = 724.52 \text{ N}$$

For F.B.D. of *A*,

$\sum F_y = 0$:

$$N_A + P \sin 30 - 500 = 0$$

$$\therefore \quad N_A = 500 - P \sin 30$$

$\sum F_x = 0$:

$$P \cos 30 - T - 0.2\, N_A = 0$$

$$P \cos 30 - 724.52 - 0.2\,(500 - P \sin 30) = 0$$

$$P\,(\cos 30 + 0.2 \sin 30) = 724.52 + 0.2 \times 500$$

$$\therefore \quad \boxed{P = 853.52 \text{ N}}$$

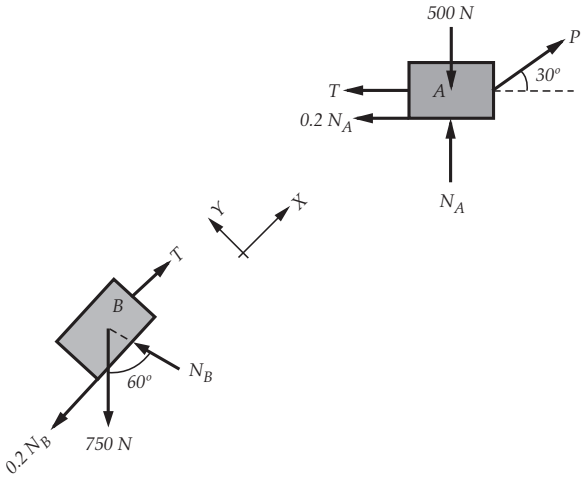


Fig. 7.5 (a)

Important Multiple Choice Questions

Q.1 *Angle of friction is angle between _____ .*

a) the incline and horizontal

b) the normal reaction and friction force

c) the weight of the body and the friction force

d) normal reaction and the resultant

VTU : Feb.-09, Aug.-10, 11, Dec.-11, Jan-13, June-13

(Ans. : d)

Q.2 *The force of friction developed at the contact surface is always _____ .*

a) parallel to the plane and along the direction of the applied force.

b) perpendicular to the plane

c) parallel to the plane and opposite to the direction of motion.

d) all the above

VTU : Feb.-09, June-13

(Ans. : c)

Q.3 *The maximum inclination of the plane on which the body free from external forces can repose is called _____ .*

a) cone of friction *b) angle of friction*

c) angle of repose *d) none of the above*

VTU : Feb.-09, June-13

(Ans. : c)

Q.4 *The force of friction depends on _____ .*

a) area of contact *b) roughness of the surfaces*

c) both area of contact and roughness of the surfaces

d) none of the above

VTU : Feb.-09, Dec.-11, June-12, 13, Jan-13

(Ans. : b)

Q.5 *The maximum frictional force developed when a body just begins to slide over the surface of another body is _____ .*

a) sliding friction *b) rolling friction* *c) limiting friction* *d) none*

VTU : Aug.-09, Jan.-13

(Ans. : c)

Q.6 *The angle which an inclined surface makes with the horizontal when a body placed on it is on the verge of moving down is called _____ .*

a) angle of repose *b) angle of friction*

c) angle of inclination *d) none*

VTU : Aug.-09, 11, Feb.-11

(Ans. : a)

Q.7 *Frictional force is independent of _____ .*

a) Coefficient of friction *b) angle of friction*

c) Shape and size of surface of contact *d) none of the above*

VTU : Aug.-09

(Ans. : c)

Q.8 *Compared to static friction, kinetic friction is _____ .*

a) greater *b) smaller* *c) very large* *d) zero*

VTU : Aug.-09, Dec.-11, June-12, Jan.-13, Dec.-13

(Ans. : b)

Q.9 *Coefficient of friction (μ) is given by _____ .*

a) $\mu = F \cdot R$ *b)* $\mu = \frac{F}{R}$ *c)* $\mu = \frac{R}{F}$ *d)* $\mu = F^2$

VTU : Feb.-10
(Ans. : b)

Q.10 *If* ϕ *= angle of friction and* μ *= coefficient of friction, then which equation is valid?*

a) $\tan \phi = \mu$ *b)* $\tan\phi = \frac{1}{\mu}$ *c)* $\sin\phi = \mu$ *d)* $\cos\phi = \mu$

VTU : Feb.-10, Aug.-11
(Ans. : a)

Q.11 *If* ϕ *= angle of friction and* α *= angle of repose then which relation is correct.*

a) $\phi = \frac{1}{\alpha}$ *b)* $\phi = \alpha$ *c)* $\phi = \tan\alpha$ *d)* $\alpha = \tan\phi$

VTU : Feb.-10, Dec.-11
(Ans. : b)

Q.12 *Force of friction developed at contact surface is _____ .*

a) opposite to the direction of motion

b) along the direction of motion *c) perpendicular to plane*

d) all of these

VTU : Feb.-10
(Ans. : a)

Q.13 *When a body resting on a rough plane is acted upon by gradually increasing tangential force, _____.*

a) frictional force is zero. *b) frictional force remains constant.*

c) frictional force increases indefinitely.

d) there is a limit up to which frictional force can increase.

(Ans. : d)

Q.14 *Frictional force acts _____ the surfaces in contact.*

a) tangential to *b) normal to* *c) inclined to* *d) away from*

VTU : June-12, Dec.-13
(Ans. : a)

Q.15 *Coulomb's laws of friction can be applied to _____.*

a) fluid friction *b) fluid-structure interaction*

c) dry friction between solid bodies *d) lubricated surfaces.*

(Ans. : c)

Q.16 *Impending motion of a body refers to a _____.*

a) body at rest. *b) body about to move.*

c) body moving with uniform speed. *d) body moving with uniform acceleration.*

(Ans. : b)

Q.17 *At the point of impending motion, the static frictional force is _____.*

a) zero *b) maximum* *c) minimum* *d) infinite*

VTU : June-2012

(Ans. : b)

Q.18 *At the point of impending motion _____.*

a) the body is on the verge of moving.

b) the frictional force reaches the maximum value.

c) the frictional force is equal to the tangential applied force.

d) all of these.

(Ans. : d)

Q.19 *Force required to start motion is _____.*

a) less than that required for keeping it in motion.

b) more than that required for keeping it in motion.

c) same as the force required for keeping it in motion.

d) zero, while force required for keeping it in motion is non-zero.

(Ans. : b)

Q.20 *The tangent of the angle of friction is _____.*

a) angle of repose *b) co-efficient of friction*

c) cone of friction *d) limiting friction*

VTU : Feb.-11, Dec.-13

(Ans. : b)

Q.21 *The angle made by the resultant of normal reaction and frictional force with the normal reaction at the point of impending motion is called _____.*

a) angle of inclination *b) angle of repose*

c) angle of friction *d) normal angle.*

(Ans. : c)

Q.22 *The unit of co-efficient of friction is _____.*

a) newton *b) radian* *c) metre* *d) dimensionless*

(Ans. : d)

Q.23 *When a block of weight W resting on a rough inclined plane of inclination θ does not slide, then the frictional force acting on it is _____*

a) W sin θ *b) W cos θ* *c) μ W sin θ* *d) μ W cos θ.*

(Ans. : a)

Q.24 *A body just begins sliding on a horizontal surface when a force of 100 N is applied parallel to the surface. If the body is pulled with a horizontal force of 50 N, the frictional force will be _____.*

a) zero *b) 100 N* *c) 50 N* *d) 5 N*

(Ans. : c The applied force is less than the force of limiting friction.)

Q.25 *When a bicycle is in motion and is not being pedalled, the frictional force exerted by the ground on the bicycle is _____.*

a) in the backward direction on the front wheel and in the forward direction on the rear wheel

b) in the forward direction on the front wheel and in the backward direction on the rear wheel

c) in the backward direction on both the front and the rear wheels

d) in the forward direction on both the front and the rear wheels

(Ans. : c)

Q.26 *When a bicycle is in motion and is being pedalled, the frictional force exerted by the ground on the bicycle is _____.*

a)in the backward direction on the front wheel and in the forward direction on the rear wheel

b)in the forward direction on the front wheel and in the backward direction on the rear wheel

c)in the backward direction on both the front and the rear wheels

d)in the forward direction on both the front and the rear wheels

(Ans. : a)

Q.27 *A man is moving towards right. The direction of frictional force exerted by the ground on the man's feet is in _____.*

a) backward *b) forward* *c) upward* *d) downward*

(Ans. : b)

Q.28 *If a man moving towards right exerts a normal force of 100 N on the ground and* μ_s *= 0.3 and* μ_k *= 0.2, the frictional force exerted on the man's feet by the ground is _____.*

a) 20 N towards left *b) 20 N towards right*

c) 30 N towards left *d) 30 N towards right*

(Ans. : d)

Q.29 *A block of weight 100 N is placed on a plane surface inclined at 20° with the horizontal. If* μ_s *= 0.3 and* μ_k *= 0.2, the block will _____.*

a) slide down with constant velocity *b) remain stationary*

c) slide down with constant acceleration *d) move upward*

(Ans. : c

Angle of repose is $\theta = \tan^{-1} \mu_s$

$\therefore$ $\theta = \tan^{-1} 0.3 = 16.7°$

As the angle of inclination is greater than the angle of repose, block slides down)

Q.30 *The frictional force on the block in Q.29 is _____.*

a) 18.8 N upward along the incline *b) 18.8 N downward along the incline*

c) 28.2 N upward along the incline *d) 28.2 N downward along the incline*

(Ans. : a
$F_r = \mu_k N = 0.2 \times 100 \cos 20$
$\therefore F_r = 18.8$ N up the incline.)

Q.31 *A block of weight 100 N is placed on a plane surface inclined at at 10 °; with the horizontal. If $\mu_s = 0.3$ and $\mu_k = 0.2$, the block will _____.*

a) slide down with constant velocity *b) remain stationary*

c) slide down with constant acceleration *d) move upwards*

(Ans. : b

Angle of inclination is less than the angle of repose.

Q.32 *The frictional force acting on the block in Q.31 is _____.*

a) $0.3 \times 100 \cos 30$ N *b) $0.2 \times 100 \cos 30$ N* *c) 100 sin 10 N* *d) 0*

(Ans. : c

As the block is stationary,

$F_r = w \sin \theta$

$= 100 \sin 10$

Q.33 *A block of weight 50 N is kept on a wooden plank and the inclination of the plank with the horizontal is slowly increased. If $\mu_s = 0.2$ and $\mu_k = 0.1$, the angle at which the block starts sliding down is _____.*

a) $\tan^{-1} 0.2$ *b) $\tan^{-1} 0.1$* *c) $\sin^{-1} 0.2$* *d) $\sin^{-1} 0.1$*

(Ans. : a)

Q.34 *A block of weight 100 N is kept on a plane surface inclined at 10° to the horizontal. If $\mu_s = 0.3$ and $\mu_k = 0.2$, the force P applied parallel to the plane, required to keep the block static is _____.*

a) 75.98 N *b) 67.32 N* *c) 50 N* *d) 0*

(Ans. : d

The angle of repose is $\tan^{-1} 0.3 = 16.7°$.

As angle of inclination is less than angle of repose,

the block doesn't slide.

$\therefore$ P = 0

Q.35 *The frictional force on the block in Q.34 is _____.*

a) 25.98 N *b) 17.36 N* *c) 50 N* *d) 0*

(Ans. : b)

Q.36 *The force P applied parallel to the plane required to move the block upwards along the plane in Q.34 is _____.*

a) 46.9 N *b) 37.1 N* *c) 100 N* *d) 50 N*

(Ans. : a) $F_r = 100 \sin 10 + 0.3 \times 100 \cos 10$

$= 46.9$ N

Q.37 *A block of mass 50 kg is kept on a plane surface inclined at an angle of 30° with horizontal. If* $\mu_s = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ *and the block is given an initial velocity of 1 m/s, the block will _____.*

a) slide down with constant velocity of 1 m/s

b) slide down with constant acceleration

c) decelerate and stop after some time

d) none of the above

(Ans. : a)

Q.38 *A right circular cylinder of weight 50 N, radius of base 25 mm and height 50 mm rests on a horizontal surface. If* $\mu_s = 0.3$ *and* $\mu_k = 0.2$*, the force required to slide the cylinder is _____.*

a) 10 N *b) 15 N* *c) 20 N* *d) 25 N*

(Ans. : (b) $P = \mu_s W = 0.3 \times 50 = 15$ N)

Q.39 *If a force P is applied horizontally at the top of the cylinder in Q.38, the value of P at which the cylinder topples is _____.*

a) 10 N *b) 15 N* *c) 20 N* *d) 25 N*

(Ans. : d From Fig. 7.6,

$\sum M_A = 0$:

$50 \times 25 - P \times 50 = 0$

$\therefore$ **P = 25 N)**

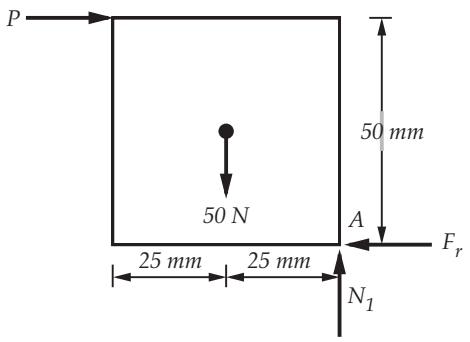


Fig. 7.6

Q.40 *A block of weight 100 N is kept on a rough horizontal surface for which* $\mu_s = 0.3$ *and* $\mu_k = 0.2$*. If a horizontal force of 20 N is applied to the block, the magnitude of frictional force is _____.*

a) 20 N *b) 30 N* *c) 40 N* *d) 0*

(Ans. : a)

Q.41 *If in Q.40, the horizontal force has a magnitude of 40 N, the magnitude of frictional force will be _____.*

a) 20 N *b) 30 N* *c) 40 N* *d) 0*

(Ans. : a)

Q.42 *In Q.40, the block _____.*

a) slides with constant velocity *b) slides with constant acceleration*

c) remain static *d) any one of the above*

(Ans. : c)

Q.43 *In Q.41, the block _____.*

a) slides with constant velocity *b) slides with constant acceleration*

c) remains static *d) any one of the above*

(Ans. : b)

Q.44 *A right circular cylinder of weight 50 N, radius of base 25 mm and height 50 mm rests on a horizontal surface. If μ_s = 0.3 and μ_k = 0.2 and a horizontal force of 20 N is applied at the top of the cylinder, it _____.*

a) slides *b) topples*

c) slides and topples *d) neither slides nor topples*

(Ans. : a)

Q.45 *If the force applied is 10 N in Q.44, the cylinder _____.*

a) slides *b) topples*

c) slides and topples *d) neither slides nor topples*

(Ans. : d)

Q.46 *A right circular cylinder of weight 100 N, radius of base 50 mm and height 250 mm rests on a horizontal surface. A horizontal force P is applied at the top of the cylinder. If μ_s = 0.3 and μ_k = 0.2, the magnitude of P for which the cylinder slides is _____.*

a) 20 N *b) 30 N* *c) 40 N* *d) 50 N*

(Ans. : b)

Q.47 *The magnitude of P for which the cylinder in Q.46 topples is _____.*

a) 20 N *b) 30 N* *c) 40 N* *d) 50 N*

(Ans. : a)

Q.48 *If the magnitude of P in Q.46 is 25 N, the maximum height at which it can be applied so that toppling of the cylinder is prevented is _____.*

a) 250 mm *b) 200 mm* *c) 150 mm* *d) 100 mm*

(Ans. : d)

Q.49 *A right circular cylinder of weight 100 N, radius of base 50 mm and height 200 mm is kept on a plane surface inclined at an angle of 30° with the horizontal. If μ_s = 0.7, the cylinder _____.*

a) slides down with constant velocity

b) slides down with constant acceleration *c) overturns*

d) neither slides nor overturns

(Ans. : c)

Q.50 *If height of the cylinder in Q.49 is 150 mm, the cylinder _____.*

a) slides down with constant velocity

b) slides down with constant acceleration *c) overturns*

d) neither slides nor overturns

(Ans. : d)

Q.51 *A right circular cylinder of weight 100 N, radius of base 50 mm and height 'h' is kept on plane surface inclined at an angle of 30° with the horizontal. If $\mu_s = 0.7$, the minimum height of the cylinder for which it overturns is _____.*

a) $\frac{50}{\sqrt{3}}$ *mm* *b)* $50\sqrt{3}$ *mm* *c)* $\frac{100}{\sqrt{3}}$ *mm* *d)* $100\sqrt{3}$ *mm*

(Ans. : b)

Q.52 *A 100 N block is pulled on a rough horizontal surface by a 30 N force with a constant velocity of 2 m/s. The frictional force on the block is _____.*

a) 100 N *b) 70 N* *c) 30 N*

d) data is not sufficient to find the frictional force

(Ans. : c)

Q.53 *A heavy block is to be supported against a vertical wall by inserting a wedge under it. The direction of frictional force on the wedge from the wall is _____.*

a) upward *b) downward* *c) horizontal*

d) inclined at 45° with horizontal

(Ans. : a)

Q.54 *If the block in Q.53 is to be raised upward, the direction of frictional force on the block from the wall is _____.*

a) upward *b) downward*

c) horizontal *d) inclined at 45° with horizontal*

(Ans. : b)

Q.55 *The angle of friction is given by _____ .*

a) $\phi = \sin^{-1}(\mu)$ *b)* $\phi = \cos^{-1}(\mu)$

c) $\phi = \tan^{-1}(\mu)$ *d)* $\phi = \cot^{-1}(\mu)$

(Ans. : c)

Q.56 *When a body resting on a rough horizontal surface is subjected to a horizontal force and the body remains at rest, the frictional force F_r = _____.*

a) $\mu_s N$ *b)* $\mu_k N$ *c) applied force* *d) none of the above*

(Ans. : c)

Q.57 *If the force applied on the object to just move it is kept constant, the object _____.*

a) moves with constant velocity *b) retards*

c) accelerates *d) none of the above*

(Ans. : c)

Q.58 *If an object is moving with constant velocity then the frictional force is ____.*

a) $\mu_s N$ *b)* $\mu_k N$

c) equal to applied force *d) both (b) and (c)*

(Ans. : d)

Q.59 *We cannot move on a perfectly smooth surface due to _____.*

a) absence of normal reaction *b) absence of friction*

c) both (a) and (b) *d) none of the above*

(Ans. : b)

Q.60 *A heavy trolley with wheels can be pushed more easily compared to a box of the same weight as _____.*

a) area of contact with the ground is less for the trolley

b) normal reaction on the wheels is less

c) rolling friction is less than sliding friction *d) none of the above*

(Ans. : c)

Q.61 *The frictional force is independent of _____ .*

a) the area of contact *b) the coefficient of friction*

c) the normal reaction *d) the angle of friction* **VTU : Aug.-10**

(Ans. : a)

Q.62 *Once a body just begins to slide, it continues to slide because _____ .*

a) the body has inertia *b) inertia force acts on the body*

c) the body accelerates *d) the frictional force becomes less*

VTU : Aug.-10

(Ans. : d)

Q.63 *The coefficient of friction between two surfaces is the constant of proportionality between the applied tangential force and the normal reaction _____ .*

VTU : Aug.-10

a) at the instant of application of the force *b) at any instant when the body is at rest*

c) at the instant of impending motion *d) at an instant after the motion takes place.*

(Ans. : c)

Q.64 *If the angle of friction is zero, a body will experience _____ .* **VTU : Feb.-11**

a) infinite friction *b) zero friction* *c) the force of friction normal to the plane*

d) the force of friction in the direction of motion.

(Ans. : b)

Q.65 *The coefficient of friction depends on _____ .*

a) area of contact *b) shape of the surface*

c) strength of the surface *d) none of these.* **VTU : Feb.-11**

(Ans. : d)

Q.66 *A body of weight Q is placed on an inclined rough plane. The inclination of the plane with the horizontal is less than the angle of friction. The body will*

a) be in motion b) move downwards c) be in equilibrium d) move upwards

VTU : Aug.-11

(Ans. : c)

Q.67 *Which of the following statements is correct ?*

a) Friction always opposes motion

b) Friction can be in the direction of motion

c) Friction cannot be in the direction of motion d) None of the above.

(Ans. : b)

Q.68 *The equation* $F_r = \mu_s N$ *can be used when* _____.

a) object is moving *b) object is static*

c) motion is impending *d) none of the above*

(Ans. : c)

Q.69 *The maximum frictional force is* _____.

a) $\mu_s N$ *b)* $\mu_k N$

c) depends on applied force *d) none of the above*

(Ans. : a)

Q.70 *Friction is required to* _____.

a) produce motion *b) stop motion*

c) both a) and b) *d) none of the above*

(Ans. : c)

Q.71 *A block of weight W is held against a vertical wall by applying a horizontal force F. The minimum value of F needed to hold the block is* _____.

a) $\mu_s W$ *b)* $\frac{W}{\mu_s}$ *c)* $\mu_k W$ *d) none of the above*

(Ans. : b)

Q.72 *For a perfectly smooth surface, the coefficient of static friction is*_____.

a) 1 *b) < 1* *c) >1* *d) 0*

(Ans. : d)

Q.73 *For a perfectly smooth surface, the coefficient of kinetic friction is*_____.

a) 1 *b) < 1* *c) > 1* *d) 0*

(Ans. : d)

Q.74 *Friction can be reduced by* _____ .

a) use of lubricants *b) making surfaces smooth*

c) using ball bearings *d) any of the above*

(Ans. : d)

Q.75 *Minimum force required to move an object on an inclined plane is _____.*

a) same as maximum force for static equilibrium

b) zero *c) equal to force of kinetic friction*

d) none of the above

(Ans. : a)

Q.76 *Friction force is a _____ force.* **VTU : Dec.-13**

a) active *b) passive* *c) normal* *d) none of these*

(Ans. : b)

❑❑❑

Notes

8 Centroid

Chapter at a Glance

- For line elements,

$$\overline{X} = \frac{\int x\,dl}{l}$$

$$\overline{Y} = \frac{\int y\,dl}{l}$$

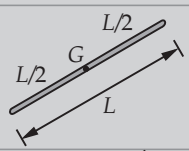
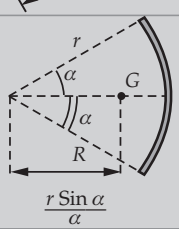
- For areas,

$$\overline{X} = \frac{\int x\,dA}{A}, \quad \overline{Y} = \frac{\int y\,dA}{A}$$

- For a composite body made up of line segments,

$$\overline{X} = \frac{\sum x\,l}{\sum l} = \frac{x_1 l_1 + x_2 l_2 + \ldots}{l_1 + l_2 + \ldots}$$

$$\overline{Y} = \frac{\sum y\,l}{\sum l} = \frac{x_1 l_1 + y_2 l_2 + \ldots}{l_1 + l_2 + \ldots}$$

Sr. No.	Basic line segment		Length	Location of centroid
	Description	Figure		
1.	Straight line		L	Centre of the line.
2.	Arc of a circle of radius r and inscribed angle 2α		$2\alpha r$	On the line of symmetry, at a distance of $\frac{r \sin\alpha}{\alpha}$ from the centre.

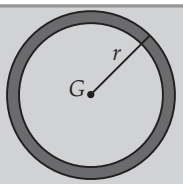
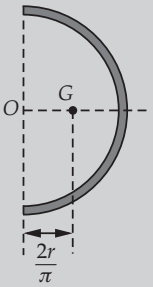
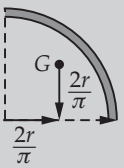
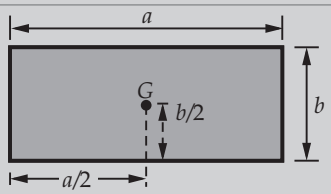
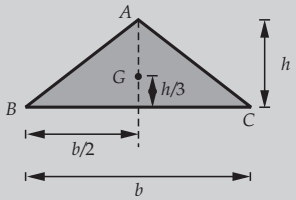
3.	Circle of radius r		$2\pi r$	Centre of the circle.
4.	Semicircular arc of radius r		πr	On the line of symmetry at a distance of $\frac{2r}{\pi}$ from the centre.
5.	Quarter circular arc of radius r		$\frac{\pi r}{2}$	A distance $\frac{2r}{\pi}$ from centre along one radius and then $\frac{2r}{\pi}$ from that point perpendicular to that radius.

Table 8.1 Centroids of line segments

- For a composite plane lamina,

$$\overline{X} = \frac{\sum x A}{\sum A} = \frac{x_1 A_1 + x_2 A_2 + \ldots}{A_1 + A_2 + \ldots}$$

$$\overline{Y} = \frac{\sum y A}{\sum A} = \frac{y_1 A_1 + y_2 A_2 + \ldots}{A_1 + A_2 + \ldots}$$

Sr. No.	Basic area		Area	Location of centroid
	Description	Figure		
1.	Rectangle of dimensions $a \times b$		ab	Distance $\frac{a}{2}$ along the side of dimension a and from that point, distance $\frac{b}{2}$ in a perpendicular direction.
2.	Isosceles triangle of base b, height h, $AB = AC$		$\frac{1}{2} bh$	On the line of symmetry at height $\frac{h}{3}$ from the base.

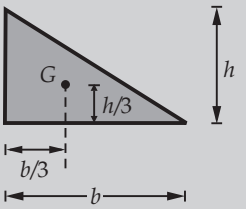
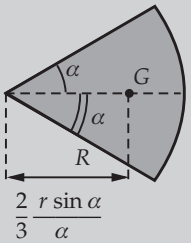
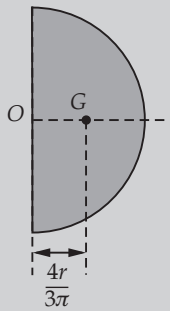
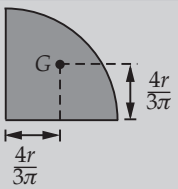
3.	Right angled triangle of base b, height h.		$\frac{1}{2}bh$	From the vertex where there is right angle, at a distance of $\frac{1^{rd}}{3}$ of one side and then $\frac{1^{rd}}{3}$ of other side in a perpendicular direction.
4.	Sector of radius r and angle 2α.		$r^2\alpha$	On the line of symmetry, at distance $\frac{2}{3}\frac{r\sin\alpha}{\alpha}$ from the centre.
5.	Semi circular plane lamina of radius r.		$\frac{\pi r^2}{2}$	On the line of symmetry at distance $\frac{4r}{3\pi}$ from the centre.
6.	Quarter circular plane lamina of radius r.		$\frac{\pi r^2}{4}$	A distance $\frac{4r}{3\pi}$ from centre along one radius and then $\frac{4r}{3\pi}$ from that point perpendicular to that radius.
7.	Circle of radius r.		πr^2	Centre of circle.

Table 8.2 Centroids of plane laminae

Method to Find Centroid by Integration

1) Choose appropriate co-ordinate system depending upon symmetry of the problem.
2) Choose a suitable differential element. For lines, use differential element of length dL. For areas choose differential element of area dA.
3) Determine the co-ordinates of the centroid of the differential element.
4) Evaluate the integrals to find co-ordinates of centroid.

Method to Find Centroid of Composite Bodies

1) Choose an appropriate origin and a co-ordinate system.
2) Divide the body into basic shapes illustrated in Table 8.1 and 8.2. If any of the basic shape has to be removed, treat its area or length to be negative.
3) Locate the centroid of each basic shape as illustrated in Table 8.1 and 8.2.
4) Write the co-ordinates of centroids of basic shapes with respect to the chosen origin as $(x_1, y_1), (x_2, y_2)$ etc.
5) Use appropriate formulae to find $\overline{X}$ and $\overline{Y}$.
6) In problems of finding some length when location of centroid is given, take moment about an axis passing through the centroid. The total moment of all the areas or lengths will be zero. For moment about vertical axis, choose sign convention as in suspended objects. For moment about a horizontal axis, we can consider the moment due to area or length above the axis as positive and below the axis as negative.

Important Theory Question and Answer

Q.1 *Distinguish between centroid and centre of gravity.* **VTU : Aug.-07, Feb.-08**

Ans : 1) Centroid is the geometrical centre of the object whereas centre of gravity is the point through which the weight of the body acts.

2) Centroid is applicable to bodies having uniform density whereas centre of gravity is applicable to bodies having uniform as well as non - uniform density.

3) Centroid and centre of gravity do not coincide for very large objects like mountains as the gravitational acceleration varies with height.

Important Solved Examples

Example 8.1 *Determine the Y-co-ordinate of centroid of the triangular area of base b and height h shown in Fig. 8.1 from first principles.* **VTU : Feb.-03, 09, 13, June-13**

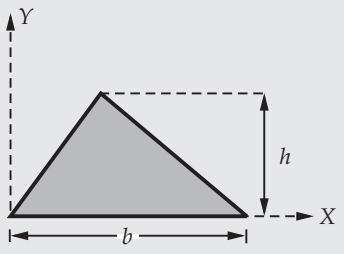


Fig. 8.1

Solution : Consider a horizontal strip of length x and width dy as shown in Fig. 8.1 (a).

The area of differential element which is approximately a rectangle, is

$$dA = x\,dy$$

Using similarity of triangles,

$$\frac{x}{h-y} = \frac{b}{h}$$

$$\therefore \quad x = \frac{b\,(h-y)}{h}$$

$$\therefore \quad dA = \frac{b\,(h-y)}{h}\,dy$$

$$\therefore \quad A = \int_0^h \frac{b\,(h-y)}{h}\,dy$$

$$= \int_0^h \left(b - \frac{by}{h}\right)dy = \left[by - \frac{by^2}{2h}\right]_0^h$$

$$= bh - \frac{bh^2}{2h} = bh - \frac{bh}{2}$$

$$\therefore \quad A = \frac{1}{2}bh$$

$$\overline{Y} = \frac{\int y\,dA}{A} = \frac{\int_0^h y \cdot \frac{b}{h}(h-y)\,dy}{\frac{1}{2}bh}$$

$$= \frac{\frac{b}{h}\left[h\frac{y^2}{2} - \frac{y^3}{3}\right]_0^h}{\frac{1}{2}bh} = \frac{\frac{b}{h}\left[\frac{h^3}{2} - \frac{h^3}{3}\right]}{\frac{1}{2}bh} = \frac{\frac{b}{h}\cdot\frac{h^3}{6}}{\frac{1}{2}bh}$$

$$\therefore \quad \boxed{\overline{Y} = \frac{h}{3}}$$

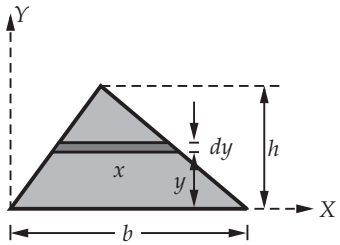


Fig. 8.1 (a)

Example 8.2 *Determine the centroid of a semi-circular area of radius r from first principles.*

VTU : Feb.-05, 06, Aug.-03, 04, 08, 11, Dec.-11,13, Marks 8

Solution : Consider a semicircle as shown in Fig. 8.2. As the x-axis is the axis of symmetry.

$$\overline{Y} = 0$$

Consider an elementary area of inscribed angle $d\theta$ as shown in Fig. 8.2.

As $d\theta$ is very small, the area is a triangle having its centroid at distance $\frac{2r}{3}$ from O.

The area of the triangular element is

$$dA = \frac{1}{2}(r)(r\,d\theta)$$

$$\therefore \quad dA = \frac{1}{2} r^2\, d\theta$$

$$\therefore \quad A = \int_{-\pi/2}^{\pi/2} \frac{1}{2} r^2\, d\theta = \frac{1}{2} r^2\, [\theta]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2} r^2 \left[\frac{\pi}{2} + \frac{\pi}{2}\right]$$

$$\therefore \quad A = \frac{\pi r^2}{2}$$

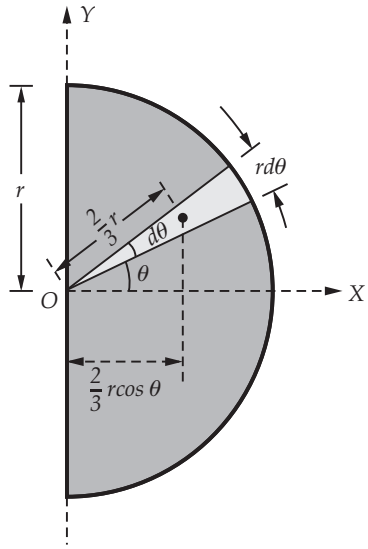


Fig. 8.2

The x - co-ordinate of centroid of the triangular elementary area is $\frac{2}{3} r \cos\theta$.

Using $\overline{X} = \dfrac{\int x\,dA}{A}$,

$$\overline{X} = \frac{\int_{-\pi/2}^{\pi/2} \left(\frac{2}{3} r\cos\theta\right)\left(\frac{1}{2} r^2\, d\theta\right)}{\left(\frac{\pi r^2}{2}\right)}$$

$$= \frac{\frac{r^3}{3} \int_{-\pi/2}^{\pi/2} \cos\theta\, d\theta}{\left(\frac{\pi r^2}{2}\right)} = \frac{2r}{3\pi} [\sin\theta]_{-\pi/2}^{\pi/2}$$

$$= \frac{2r}{3\pi}\left[\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right]$$

$$= \frac{2r}{3\pi}[1-(-1)]$$

$$\therefore \quad \boxed{\overline{X} = \frac{4r}{3\pi}}$$

The centroid of a semicircle lies on the line of symmetry at a distance of $\frac{4r}{3\pi}$ from the centre.

Example 8.3 *Locate the centroids of the following area. Refer Fig. 8.3.*

VTU : Feb.-04, Marks 10, Dec.-11, Marks 8

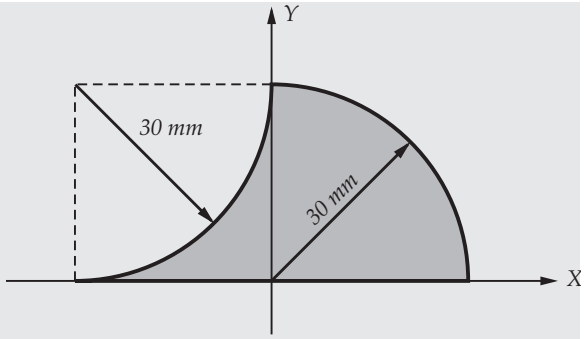


Fig. 8.3

Solution : The shaded portion in the figure can be obtained by subtracting the area of quarter circle (2) from the square (1) in second quadrant and then adding the area of quarter circle (3) in first quadrant.

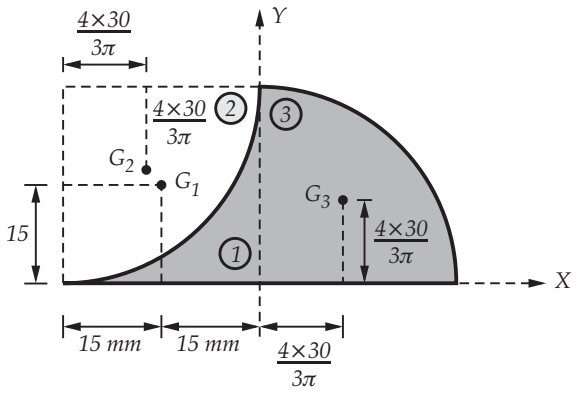


Fig. 8.3 (a)

The calculations are tabulated as follows :

Component No.	Component area $A(mm^2)$	x (mm)	y (mm)
1.	(30) (30)	– 15	15
2.	$\frac{-\pi (30)^2}{4}$	$-\left(30-\frac{4\times30}{3\pi}\right)$	$30-\frac{4\times 30}{3\pi}$
3.	$\frac{\pi \times 30^2}{4}$	$\frac{4\times 30}{3\pi}$	$\frac{4\times30}{3\pi}$

$$\sum A = 900\ mm^2$$

$$\sum Ax = 7705.75\ mm^3$$

$$\sum Ay = 10294.25\ mm^3$$

$$\overline{X} = \frac{\sum Ax}{\sum A} = \frac{7705.75}{900}$$

$$\therefore \quad \overline{X} = 8.562\ mm$$

$$\overline{Y} = \frac{\sum Ay}{\sum A} = \frac{10294.25}{900}$$

$$\therefore \quad \boxed{\overline{Y} = 11.438 \text{ mm}}$$

Example 8.4 *Locate centroid of lamina shown in Fig. 8.4 with respect to point A.*

VTU : Aug.-09, June-13, Marks 10

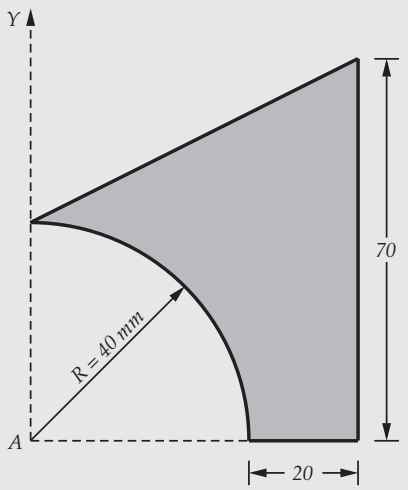


Fig. 8.4

Solution : The shaded area can be obtained by adding areas of rectangle and triangle, and then subtracting the area of quarter circle as shown in Fig. 8.4 (a).

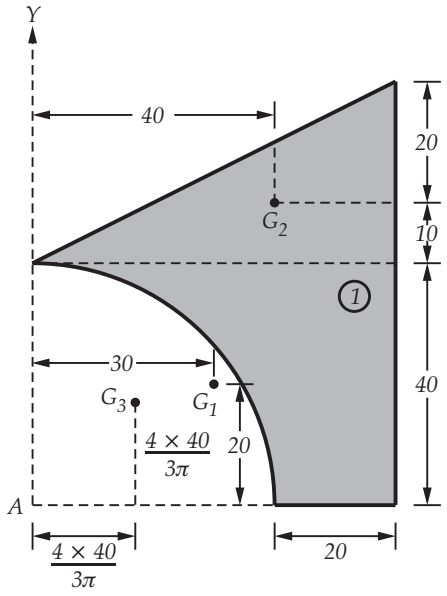


Fig. 8.4 (a)

The calculations are tabulated as follows :

Component No.	Component area $A\ (mm^2)$	x (mm)	y (mm)
1.	$(60)\ (40)$	30	20
2.	$\frac{1}{2}\ (60)\ (30)$	40	50
3.	$-\frac{\pi \times 40^2}{4}$	$\frac{4 \times 40}{3\pi}$	$\frac{4 \times 40}{3\pi}$

$$\sum A = 2043.36\ mm^2$$

$$\sum Ax = 86666.67\ mm^3$$

$$\sum Ay = 71666.67\ mm^3$$

$$\overline{X} = \frac{\sum Ax}{\sum A} = \frac{86666.67}{2043.36}$$

$$\therefore \quad \boxed{\overline{X} = 42.41\ mm}$$

$$\overline{Y} = \frac{\sum Ay}{\sum A} = \frac{71666.67}{2043.36}$$

$$\therefore \quad \boxed{\overline{Y} = 35.07\ mm}$$

Important Multiple Choice Questions

Q.1 *Moment of total area about its centroidal axis is _____ .*

a) twice the area *b) three times the area*

c) zero *d) none of the above* **VTU : Feb.-09, 11, Aug.-11, June-12,13**

(Ans. : c)

Q.2 *The centroid of a semicircle of radius R about its centroidal axis parallel to its diametric axis is _____ .*

a) $\frac{3R}{4\pi}$ *b)* $\frac{3R}{8\pi}$ *c)* $\frac{4R}{\pi}$ *d)* $\frac{4R}{3\pi}$

VTU : Feb.-09, 11, June-12,13, Dec.-13

(Ans. : d)

Q.3 *An axis over which one half of the plane figure is just mirror image of the other half is _____ .*

a) bottom most axis of the figure *b) axis of symmetry*

c) unsymmetrical axis *d) none of the above*

VTU : Feb.-09, 10, Aug.-10, June-12 ,13, Dec.-13

(Ans. : b)

Q.4 *Centroid of a triangle with base b and depth d is _____ .*

a) $\frac{b}{3}$ *and* $\frac{d}{3}$ *b)* $\frac{b}{2}$ *and* $\frac{d}{2}$ *c)* $\frac{b}{4}$ *and* $\frac{d}{4}$ *d) none of the above*

VTU : Feb.-09

(Ans. : a)

Q.5 *Centroid of plane is the point at which _____ .*

a) volume of body is concentrated

b) surface area is assumed to be concentrated

c) weight of the body is assumed to be concentrated

d) all the above.

VTU : Aug.-09

(Ans. : b)

Q.6 *Centroid of trapezium of height 'h' and parallel sides 'a' and 'b' measured from base 'b' is at a distance of _____ .*

a) $\frac{h}{2}\left(\frac{b+2a}{a+b}\right)$ *b)* $\frac{h}{2}\left(\frac{b-2a}{a+b}\right)$ *c)* $\frac{h}{3}\left(\frac{b+2a}{a+b}\right)$ *d)* $\frac{h}{3}\left(\frac{b-2a}{a+b}\right)$

VTU : Aug.-09

(Ans. : c)

Q.7 *Centroid of quarter of circular lamina lies from diameter line at a distance of _____ .*

a) $\frac{2R}{3\pi}$ *b)* $\frac{3R}{3\pi}$ *c)* $\frac{4R}{3\pi}$ *d)* $\frac{5R}{3\pi}$

VTU : Aug.-09

(Ans. : c)

Q.8 *The centroid of a triangular lamina of height 'h' is situated at a distance _____ from its apex.*

a) $\frac{h}{3}$ *b)* $\frac{2h}{3}$ *c)* $\frac{h}{2}$ *d)* $\frac{h}{4}$

VTU : Aug.-09, 10, June-12

(Ans. : b)

Q.9 *The centroid of a triangle of height 'h' is located at a distance _____ from its base.*

a) $\frac{h}{2}$ *b)* $\frac{2h}{3}$ *c)* $\frac{h}{3}$ *d)* h

VTU : Feb.-10

(Ans. : c)

Q.10 *If the given plane figure is symmetrical about vertical Y-Y axis, the centroid lies on _____.*

a) X-axis *b) Y-Y axis* *c) bottom* *d) top*

VTU : Feb.-10

(Ans. : b)

Q.11 *Centroid of a sector of a circle lies at a distance of _____.*

a) $\frac{4r}{3\pi}$ *b)* $\frac{2r\sin\alpha}{3\alpha}$ *c)* $\frac{r\sin\alpha}{2\alpha}$ *d) None of the above*

(Ans. : b)

Q.12 *The centre of gravity of an isosceles triangle with base (q) and sides (p) is _____ from its base.*

a) $\sqrt{\frac{4p^2 - q^2}{6}}$ *b)* $\sqrt{\frac{4p^2 + q^2}{6}}$ *c)* $\sqrt{\frac{4p^2 - q^2}{8}}$ *d)* $\sqrt{\frac{4p^2 + q^2}{8}}$.

(Ans. : a)

Q.13 *The C.G. of an equilateral triangle with each side (a) is _____ from any of the three sides.*

a) $\frac{a\sqrt{3}}{2}$ *b)* $\frac{a\sqrt{2}}{3}$ *c)* $\frac{a}{2\sqrt{3}}$ *d)* $\frac{a}{3\sqrt{2}}$

VTU : Feb.-11

(Ans. : c)

Q.14 *The centre of gravity of a semicircle lies at a distance of _____ from its base measured along the vertical radius.*

a) $\frac{3r}{4\pi}$ *b)* $\frac{4r}{3\pi}$ *c)* $\frac{4\pi}{3r}$ *d)* $\frac{3\pi}{4r}$ **(Ans. : b)**

Q.15 *The x or y co-ordinate of the centroid of a quadrant of a circular area of radius r is _____.*

a) $\frac{4r}{3\pi}$ *b)* $\frac{2r}{3\pi}$ *c)* $\frac{2r}{\pi}$ *d)* $\frac{r}{\pi}$ **(Ans. : a)**

Q.16 *The centroid of an equilateral triangle of side a with a side parallel to the X-axis is* _____.

a) $\frac{a}{2}, \frac{a}{\sqrt{6}}$ *b)* $\frac{a}{2}, \frac{a}{\sqrt{12}}$ *c)* $\frac{a}{2}, \frac{a}{\sqrt{24}}$ *d)* $\frac{a}{3}, \frac{a}{3}$

(Ans. : b)

Q.17 *The first moment of an area about the X-axis is* _____.

a) $\int x\,dA$ *b)* $\int y\,dA$ *c)* $\int x^2 dA$ *d)* $\int y^2 dA$

(Ans. : b)

Q.18 *The co-ordinates of the centroid of the right-angled triangle shown in Fig. 8.5 is* _____ .

a) $\left[\frac{b}{3}, \frac{h}{3}\right]$ *b)* $\left[\frac{2b}{3}, \frac{h}{3}\right]$

c) $\left[\frac{b}{3}, \frac{2h}{3}\right]$ *d)* $\left[\frac{2b}{3}, \frac{2h}{3}\right]$

(Ans. : c)

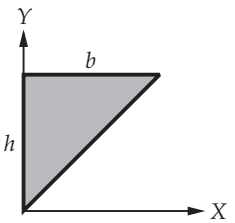


Fig. 8.5

Q.19 *The co-ordinates of the centroid of the right-angled triangle shown in Fig. 8.6 is* _____ .

a) $\left[\frac{b}{3}, \frac{h}{3}\right]$ *b)* $\left[\frac{2b}{3}, \frac{h}{3}\right]$

c) $\left[\frac{b}{3}, \frac{2h}{3}\right]$ *d)* $\left[\frac{2b}{3}, \frac{2h}{3}\right]$

(Ans. : d)

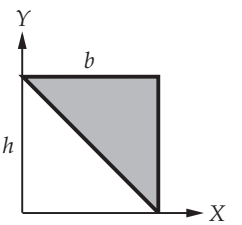


Fig. 8.6

Q.20 *Assuming a square of side a to be made up of two right-angled triangles then the distance of the centroid of each triangle with respect to the diagonal is* _____ .

a) $\frac{a}{\sqrt{2}}$ *b)* $\frac{a}{\sqrt{3}}$

c) $\frac{a}{\sqrt{9}}$ *d)* $\frac{a}{\sqrt{18}}$

(Ans. : d)

Q.21 *The x co-ordinate of centroid of the triangular plane lamina shown in Fig. 8.7 is* _____ .

a) $\frac{l}{3}$ *b)* $\frac{2l}{3}$ *c)* $\frac{l}{2\sqrt{3}}$ *d)* $\frac{l}{6}$

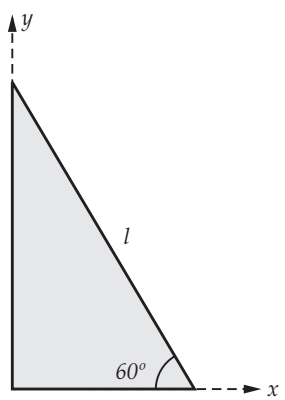


Fig. 8.7

(Ans. : d)

The length of horizontal side is $\frac{l}{2}$.

$\therefore \quad \bar{X} = \frac{l}{6}$

Q.22 *The y co-ordinate of centroid in Q.21 is _____ .*

a) $\frac{l}{3}$ *b)* $\frac{2l}{3}$ *c)* $\frac{l}{2\sqrt{3}}$ *d)* $\frac{\sqrt{3}\,l}{2}$

(Ans. : c)

The length of vertical side is $\frac{l\sqrt{3}}{2}$.

Q.23 *The x co-ordinate of centroid for the plane lamina shown in Fig. 8.8 is _____ .*

a) $\frac{5l}{6}$ *b)* $\frac{6l}{5}$

c) $\frac{2l}{3}$ *d)* $\frac{l}{3}$

(Ans. : a)

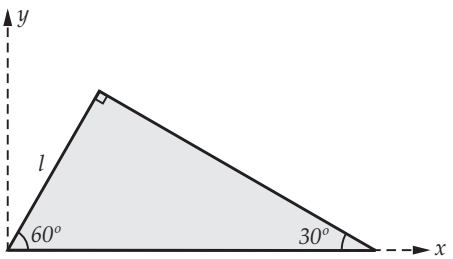


Fig. 8.8

Q.24 *The y co-ordinate of centroid in Q.23 is _____ .*

a) $\frac{l}{3}$ *b)* $\frac{2l}{3}$ *c)* $\frac{l}{2\sqrt{3}}$ *d)* $\frac{\sqrt{3}\,l}{2}$

(Ans. : c)

Q.25 *The x co-ordinate of centroid for the plane lamina shown in Fig. 8.9 is_____ .*

a) 0.654 r *b) 0.5 r* *c) 0.25 r* *d) 0.332 r*

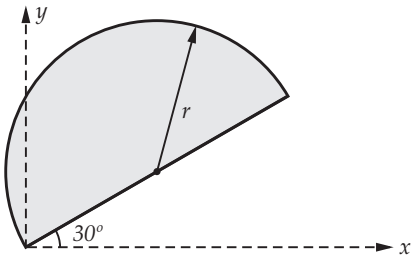


Fig. 8.9

(Ans. : a)

From Fig. 8.9 (a),

$$\bar{X} = r \cos 30 - \frac{4r}{3\pi} \cos 60$$

$$\therefore \quad \bar{X} = 0.654\ r.$$

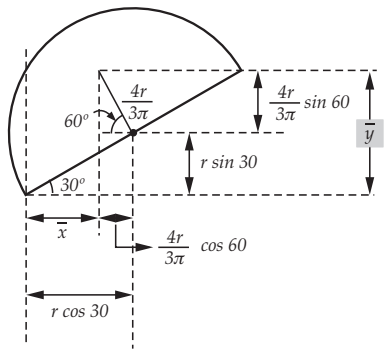


Fig. 8.9 (a)

Q.26 *The y co-ordinate of centroid in Q.25 is _____.*

a) 0.6 r *b) 0.3 r* *c) 0.87 r* *d) 0.67 r* **(Ans. : c)**

From Fig. 8.9 (a),

$\bar{Y} = r \sin 30 + \frac{4r}{3\pi} \sin 60$

$\therefore \quad \bar{Y} = 0.87\ r.$

Q.27 *If a plane lamina is suspended freely from any point on it, under equilibrium conditions, the centroid lies _____ point of suspension.*

a) horizontally to left of *b) horizontally to the right of*

c) vertically above *d) verically below* **(Ans. : c)**

Q.28 *The centroid of an isosceles triangle with base 'b' and side 'a' is _____ from its base.*

a) $\sqrt{\frac{4a^2 - b^2}{6}}$ *b)* $\sqrt{\frac{4a^2 + b^2}{6}}$ *c)* $\sqrt{\frac{4a^2 - b^2}{8}}$ *d)* $\sqrt{\frac{4a^2 + b^2}{8}}$ **(Ans. : a)**

Q.29 *The centroid of an equilateral triangle of side 'a' is at a distance _____ from any of the three sides.*

a) $\frac{a\sqrt{3}}{2}$ *b)* $\frac{a\sqrt{2}}{3}$ *c)* $\frac{a}{2\sqrt{3}}$ *d)* $\frac{a}{3\sqrt{2}}$ **(Ans. : c)**

Q.30 $\bar{X}$ *is the distance of centroid from _____ .*

a) X-axis *b) Y-axis*

c) centroidal X-axis *d) centroidal Y-axis* **(Ans. : b)**

Q.31 $\bar{Y}$ *is the distance of centroid from _____.*

a) X-axis *b) Y-axis*

c) centroidal X-axis *d) centroidal Y-axis* **(Ans. : a)**

Q.32 *Centroid of a body coincides with its centre of mass when _____.*

a) acceleration due to gravity is same for all particles of the body

b) all particles have same mass

c) body has uniform density *d) none of the above* **(Ans. : c)**

Q.33 *Centre of gravity and centre of mass of a body concide when _____.*

a) all particles have same mass *b) body has uniform density*

c) acceleration due to gravity is same for all particles of the body

d) none of the above **(Ans. : c)**

Q.34 *The centroid of a lamina _____ .*

a) must be a point on the lamina

b) is a point which can be made to lie on or outside the lamina by changing the co-ordinates system.

c) is a fixed point in space regardless of the orientation of the lamina

d) is a unique point fixed with respect to the lamina **VTU : Aug.-10**

(Ans. : d)

Q.35 *Centroid conveys some clue about _____ .*

a) the orientation of a surface *b) centre of a body*

c) shape and disposition of the area *d) area of cross-section* **VTU : Aug.-10, 11**

(Ans. : c)

Q.36 *The centre of gravity of a plane lamina will not be its geometrical centre if it is a _____ .*

a) square *b) rectangle* *c) right angled triangle* *d) equilateral triangle*

VTU : Feb.-11

(Ans. : c)

Q.37 *The first moment of area of a semicircular area about its diameter d is given by _____ .*

a) $\frac{d^3}{12}$ *b)* $\frac{d^3}{24}$ *c)* $\frac{d^3}{6}$ *d)* $\frac{d^3}{36}$

VTU : Aug.-11

(Ans. : a)

Q.38 *Centroid of a triangular area of base 'b' and height 'h' taken about an axis coincident with the base is given by*

a) $\frac{bh^3}{12}$ *b)* $\frac{b^2h}{6}$ *c)* $\frac{bh^2}{6}$ *d)* $\frac{h}{3}$ **VTU : Aug.-11**

(Ans. : d)

Q.39 *Moment of total area about its centroidal axis is _____ .*

a) twice the area *b) three times the area*

c) zero *d) area x (Centroidal distance)^2* **VTU : Dec.-11**

(Ans. : c)

Q.40 *For a steel ball of radius, R, _____ .*

a) the centroid and centre of gravity are different

b) the centroid and centre of gravity are same

c) the centroid is half the centre of gravity *d) none of these* **VTU : Dec.-11**

(Ans. : b)

Q.41 *The co-ordinates of the centroid of a quadrant of a circle of radius, r is _____ .*

a) $\bar{x} = \frac{4r}{3\pi}$, $\bar{y} = r$ *b)* $\bar{x} = r$, $\bar{y} = \frac{4r}{3\pi}$

c) $\bar{x} = \frac{4r}{3\pi}$, $\bar{y} = \frac{4r}{3\pi}$ *d)* $\bar{x} = r$, $\bar{y} = r$ **VTU : Dec.-11**

(Ans. : c)

Q.42 *If the given plane figure is symmetrical about y-y axis only, then the centroid lies on _____ .*

a) The intersection of x-x axis and y-y axis *b) x-x axis*

c) y-y axis *d) none of these* **VTU : Dec.-11**

(Ans. : c)

Q.43 *If some area is removed in a rectangle from the right side of its centroid, the centroid _____.*

a) shifts towards right *b) remains at the same point*

c) shifts towards left *d) none of these* **(Ans. : c)**

Q.44 *If some symmetric area about centroidal x - axis is removed from a rectangle, the centroid _____.*

a) shifts towards right *b) remains at the same point*

c) shifts towards left *d) none of these* **(Ans. : b)**

Q.45 *If some area is removed in a rectangle from the left side of its centroid, the centroid _____.*

a) shifts towards right *b) remains at the same point*

c) shifts towards left *d) none of these* **(Ans. : a)**

Q.46 *If a plane lamina in the shape of an equilateral triangle is made to hang from one of its vertices, the side adjacent to that vertex makes angle _____ with the horizontal.*

a) 0^o *b)* 30^o *c)* 45^o *d)* 60^o **(Ans. : d)**

Q.47 *If a plane lamina in the shape of an equilateral triangle is made to hang from one of its vertices, the side opposite to that vertex makes angle _____ with the horizontal.*

a) 0^o *b) 30^o* *c) 45^o* *d) 60^o* **(Ans. : a)**

Q.48 *If a plane lamina in the shape of an equilateral triangle is made to hang from one of its vertices, the side adjacent to that vertex makes angle _____ with the vertical.*

a) 0^o *b) 30^o* *c) 45^o* *d) 60^o* **(Ans. : b)**

Q.49 *For any plane lamina, _____.*

a) centroid and C.G are at different points

b) centroid and centre of mass are at different points

c) centroid and C.G coincide but centre of mass is different

d) centroid, C.G and centre of mass coincide **(Ans. : d)**

Q.50 *Centroid of a rectangle of base width b and depth d is _____.* **VTU : June-13**

a) b/3 and d/3 *b) b/2 and d/2* *c) b/4 and d/4* *d) none of these*

(Ans. : b)

Q.51 *Intersection of _____ number of symmetrical axes will give centroid of plane area.*

VTU : Dec.-13

a) 3 *b) 4* *c) 2* *d) none of these*

(Ans. : c)

❑❑❑

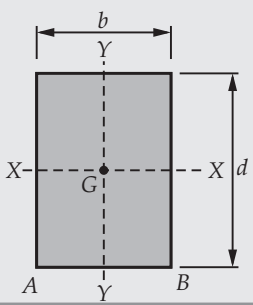
Notes

9 Moment of Inertia

Chapter at a Glance

- The moment of inertia about X-axis is
$$I_{XX} = \int y^2 dA$$
- The moment of inertia about Y-axis is
$$I_{YY} = \int x^2 dA$$
- Radius of gyration $K = \sqrt{\frac{I}{A}}$
- **Perpendicular Axes Theorem :** Moment of inertia of an area about an axis perpendicular to its plane (i.e. the polar moment of inertia) at any point is equal to the sum of moments of inertia about any two mutually perpendicular axes through the same point in the plane of the area.
- **Parallel Axes Theorem :** The moment of inertia of any area about an axis in its plane is the sum of moment of inertia about a parallel axis passing through the centroid of the area (known as centroidal axis) and the product of area and square of the distance between the two parallel axes.

Moment of Inertia of Standard Areas :

Sr. No.	Basic area		M. I.
	Description	Figure	
1.	Rectangle		$I_{XX} = \frac{bd^3}{12}$ $I_{YY} = \frac{db^3}{12}$ $I_{ZZ} = \frac{bd}{12}(b^2 + d^2)$ $I_{AB} = \frac{bd^3}{3}$

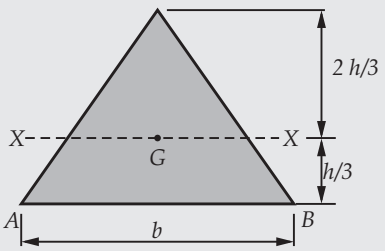
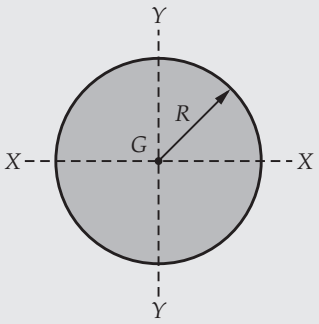
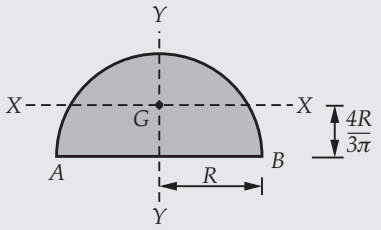
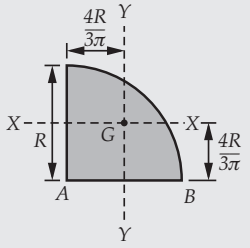
2.	Triangle		$I_{XX} = \frac{bh^3}{36}$ $I_{AB} = \frac{bh^3}{12}$
3.	Circle		$I_{XX} = I_{YY} = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}$ $I_{ZZ} = \frac{\pi R^4}{2} = \frac{\pi D^4}{32}$
4.	Semicircle		$I_{XX} = 0.11\ R^4$ $I_{YY} = \frac{\pi R^4}{8} = \frac{\pi D^4}{128}$ $I_{AB} = \frac{\pi R^4}{8} = \frac{\pi D^4}{128}$
5.	Quarter circle		$I_{XX} = I_{YY} = 0.055\ R^4$ $I_{AB} = \frac{\pi R^4}{16} = \frac{\pi D^4}{256}$

Table 9.1

Method for Solution

- The moment of inertia of composite area about centroidal x axis is

$$I_{XX} = \sum (I_X + Ar_x^2)$$

where $r_x = \overline{Y} - y$ or $r_x = y - \overline{Y}$

- The moment of inertia of composite area about centroidal y axis is

$$I_{YY} = \sum (I_Y + Ar_y^2)$$

where $r_y = \overline{X} - x$ or $r_y = x - \overline{X}$

Important Theory Questions and Answers

Q.1 *State and prove parallel axes theorem of moment of inertia.*

VTU : Aug.-07,08,11; Feb.-08,09,13, Dec.-11, June-12, 13, Marks 6

Ans. : Statement : The moment of inertia of any area about an axis in its plane is the sum of moment of inertia about a parallel axis passing through the centroid of the area (known as centroidal axis) and the product of area and square of the distance between the two parallel axes.

Proof : Consider an area A with centroid G as shwon in Fig. 9.1. To find moment of inertia about AB, consider an area element dA at distance y as shown. Let CD be an axis parallel to AB passing through the centroid G of the area A. Let d be the distance between the two parallel axes AB and CD and y_1, the distance of dA from centroidal axis. By definition,

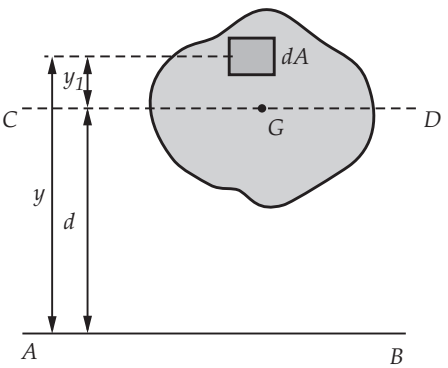


Fig. 9.1

$$I_{AB} = \int y^2 dA$$

But $y = y_1 + d$

$$\therefore \quad I_{AB} = \int (y_1 + d)^2 dA = \int (y_1^2 + 2y_1 d + d^2) dA$$

$$\therefore \quad I_{AB} = \int y_1^2 dA + 2d \int y_1 \, dA + d^2 \int dA \qquad ...(9.1)$$

The moment of inertia of the area A about centroidal axis CD is

$$I_G = \int y_1^2 dA$$

$\int y_1 dA$ is the moment of area A about its centroidal axis. As area is always symmetrical about its centroidal axis,

$$\int y_1 dA = 0$$

Also, $\int dA = A$

Substituting these in equation (9.1)

$$I_{AB} = I_G + 0 + d^2 A$$

$$\therefore \quad I_{AB} = I_G + Ad^2$$

This proves the parallel axes theorem.

Q.2 *Derive the expression for the moment of inertia of a semicircular lamina of radius (r) about its centroidal axis parallel to the diameter.* **VTU : Aug.-11, Marks 4**

Ans. : Consider a semicircle of radius R as shown in Fig. 9.2 Choose differential element of angular width $d\theta$ at angle θ and radial width dr at distance r from the centre. The area of the differential element will be

$$dA = (rd\theta)(dr)$$

$$\therefore \quad dA = rdr\, d\theta$$

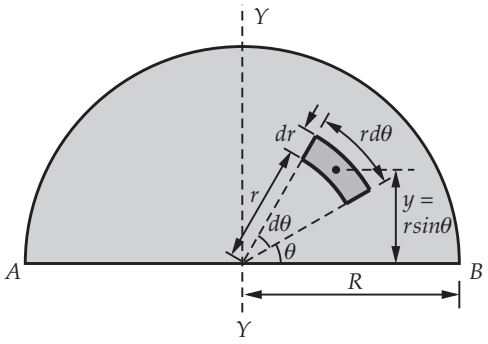


Fig. 9.2

The distance of this differential element from the diameter AB is

$$y = r\sin\theta$$

M.I. of differential element about AB is,

$$dI_{AB} = y^2 dA = (r\sin\theta)^2\,(r\,dr\,d\theta)$$

$$\therefore \quad dI_{AB} = r^3 \sin^2\theta\, dr\, d\theta$$

$$I_{AB} = \int_0^R \left[\int_0^\pi \sin^2\theta\, d\theta\right] r^3\, dr = \int_0^R \left[\int_0^\pi \left(\frac{1-\cos 2\theta}{2}\right) d\theta\right] r^3\, dr$$

$$= \frac{1}{2}\int_0^R \left[\theta - \frac{\sin 2\theta}{2}\right]_0^\pi r^3 dr = \frac{1}{2}\int_0^R \pi r^3\, dr$$

$$\therefore \quad I_{AB} = \frac{\pi R^4}{8} \qquad \ldots (9.2)$$

The centroid is at a distance of $\frac{4R}{3\pi}$ from AB.

$$I_{AB} = I_G + Ad^2$$

$$I_G = I_{AB} - Ad^2$$

$$A = \frac{\pi R^2}{2} \text{ and } d = \frac{4R}{3\pi}$$

$$\therefore \quad I_G = \frac{\pi R^4}{8} - \left(\frac{\pi R^2}{2}\right)\left(\frac{4R}{3\pi}\right)^2 = R^4\left(\frac{\pi}{8} - \frac{8}{9\pi}\right)$$

$$\therefore \quad I_G = 0.11\, R^4 \quad \ldots (9.3)$$

As $R = \frac{D}{2}$

$$I_G = 0.00686\, D^4 \quad \ldots (9.4)$$

The moment of inertia about a centroidal axis perpendicular to diameter (Y axis shown in Fig. 9.2) is

$$I_G = \frac{\pi R^4}{8} = \frac{\pi D^4}{128}$$

Important Solved Examples

Example 9.1 *Calculate the least radius of gyration for the section shown in Fig. 9.3.*

VTU : Aug.-03, Marks 14, Feb.-11, Marks 12

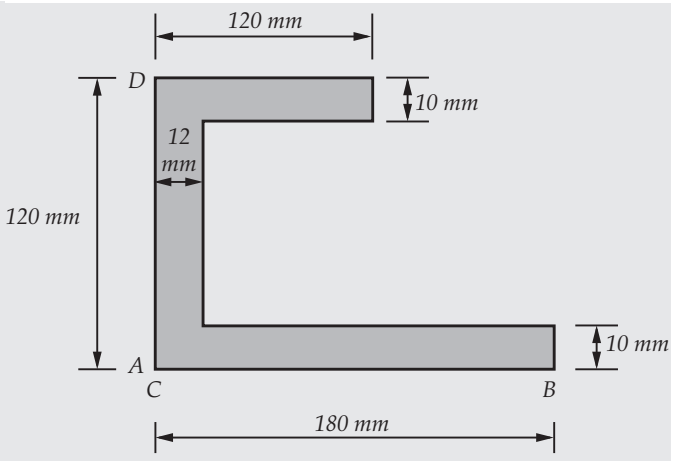


Fig. 9.3

Solution : The least radius of gyration corresponds to the least moment of inertia. We find both I_{XX} and I_{YY} and use the smaller of the two to find least radius of gyration.

To find centroid, use A as origin. Dividing the given area into three rectangles,

$$\overline{X} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

$$= \frac{(180\times10)(90) + (12\times100)(6) + (120\times10)(60)}{(180\times10) + (12\times100) + (120\times10)}$$

$$\overline{X} = 57.429 \text{ mm}$$

$$\therefore \quad \overline{Y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{(180\times10)(5) + (12\times100)(60) + (120\times10)(115)}{(180\times10) + (12\times100) + (120\times10)}$$

$$\therefore \quad \overline{Y} = 52.143 \text{ mm} .$$

The positions of centroidal axes are shown in Fig. 9.3 (a).

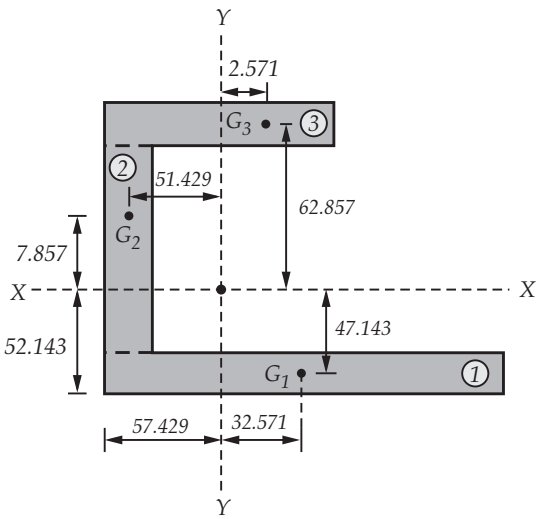


Fig. 9.3 (a)

The calculations are tabulated as follows :

Component No.	Component area A (mm^2)	x (mm)	y (mm)	I_G		$r_x = \overline{Y} - y$ (mm)	$r_y = \overline{X} - x$ (mm)
				$I_x\ (mm^4)$	$I_y\ (mm^4)$		
1.	180×10	90	5	$\frac{180\times10^3}{12}$	$\frac{10\times180^3}{12}$	52.143 – 5 = 47.143	57.429 – 90 = – 32.571
2.	12×100	6	60	$\frac{12\times100^3}{12}$	$\frac{100\times12^3}{12}$	52.143 – 60 = – 7.857	57.429 – 6 = 51.429
3.	120×10	60	115	$\frac{120\times10^3}{12}$	$\frac{10\times120^3}{12}$	52.143 – 115 = – 62.857	57.429 – 60 = – 2.571

$$I_{XX} = \sum (I_X + A\, r_x^2)$$

and $$I_{YY} = \sum (I_Y + A r_y^2)$$

$$I_{XX} = \left[\frac{180\times10^3}{12} + (180\times10)\times 47.143^2\right] + \left[\frac{12\times100^3}{12} + (12\times100)\times 7.857^2\right]$$

$$+\left[\frac{120\times10^3}{12} + (120\times10)\times 62.857^2\right]$$

$\therefore \quad I_{XX} = 9.8407\times 10^6 \ mm^4.$

$$I_{YY} = \left[\frac{10\times180^3}{12} + (10\times180)\times 32.571^2\right] + \left[\frac{100\times12^3}{12} + (100\times12)\times 51.429^2\right]$$

$$+\left[\frac{10\times120^3}{12} + (10\times120)\times 2.571^2\right]$$

$\therefore \quad I_{YY} = 11.4058\times10^6 \ mm^4$

As, $I_{XX} < I_{YY}$,

Least radius of gyration, $K_{min} = \sqrt{\frac{I_{XX}}{\sum A}}$

$\therefore \quad K_{min} = \sqrt{\frac{9.8407\times10^6}{180\times10 + 12\times100 + 120\times10}}$

$\therefore \quad \boxed{K_{min} = 48.405 \text{ mm}}$

Example 9.2 *Determine the moment of inertia and radii of gyration of the area shown in Fig. 9.4 about the base AB and the centroidal axis parallel to AB.*

VTU : Feb.-07, 09,13, Marks 14, June-12, Marks 10

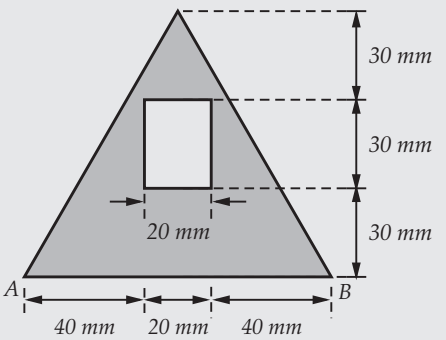


Fig. 9.4

Solution : The M.I. of triangle about its base is $\frac{bh^3}{12}$.

$$\therefore \quad I_{AB} = \frac{100\times90^3}{12} - \left[\frac{20\times30^3}{12} + (20\times30)\times 45^2\right]$$

$\therefore \quad \boxed{I_{AB} = 4.85\times 10^6 \ mm^4}$

Radius of gyration about AB is

$$K_{AB} = \sqrt{\frac{I_{AB}}{\sum A}} = \sqrt{\frac{4.85\times10^6}{\frac{1}{2}\times100\times90 - 20\times30}}$$

$\therefore$ $K_{AB} = 35.137$ mm

$$\overline{Y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{\left(\frac{1}{2}\times 100\times90\right)(30) - (20\times30)(45)}{\frac{1}{2}\times 100\times 90 - 20\times 30}$$

$\therefore$ $\overline{Y} = 27.692$ mm

Using parallel axes theorem for the complete area,

$$I_{AB} = I_{XX} + A(\overline{Y})^2$$

$\therefore$ $$I_{XX} = I_{AB} - A(\overline{Y})^2 = 4.815\times 10^6 - \left(\frac{1}{2}\times 100\times90 - 20\times30\right)\times 27.692^2$$

$\therefore$ $I_{XX} = 1.824\times 10^6\ mm^4$

$$K_{XX} = \sqrt{\frac{I_{XX}}{\sum A}} = \sqrt{\frac{1.824\times10^6}{\frac{1}{2}\times 100\times 90 - 20\times30}}$$

$\therefore$ $K_{XX} = 21.626$ mm

Example 9.3 *Determine the moment of inertia of the symmetrical I-section shown in Fig. 9.5 about its centroidal X-X and Y-Y axis.*

VTU : Dec.-13, Marks 10

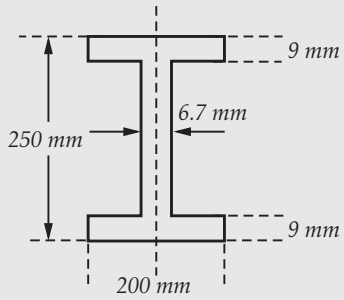


Fig. 9.5

Solution : As the given area is symmetric about horizontal and vertical lines pressing through the centre, these lines are the centroidal X-X and Y-Y axes respectively as shown in Fig. 9.5 (a).

To find I_{XX}, parallel axes theorem is required.

$$I_{XX} = \Sigma\left[I_x + A\left(\overline{Y} - y\right)^2\right]$$

$A_1 = 200 \times 9 \text{ mm}^2$, $y_1 = 4.5$ mm

$A_2 = 232 \times 6.7 \text{ mm}^2$, $y_2 = 125$ mm

$A_3 = 200 \times 9 \text{ mm}^2$, $y_3 = 245.5$ mm

$\overline{Y} = 125$ mm

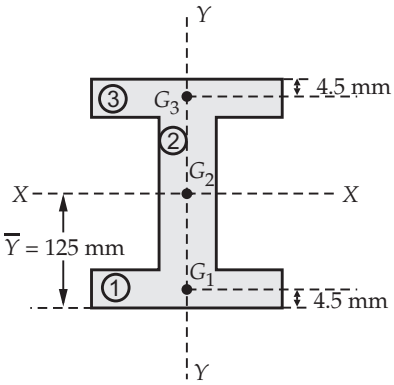


Fig. 9.5 (a)

$$\therefore I_{XX} = \left[\frac{200 \times 9^3}{12} + 200 \times 9 \times (125 - 4.5)^2\right] + \left[\frac{6.7 \times 232^3}{12} + 0\right] + \left[\frac{200 \times 9^3}{12} + 200 \times 9 \times (125 - 245.5)^2\right]$$

$$\therefore \quad \boxed{I_{xx} = 59.27 \times 10^6 \text{ mm}^4}$$

$$I_{YY} = \Sigma I_Y = \frac{9 \times 200^3}{12} + \frac{232 \times 6.7^3}{12} + \frac{9 \times 200^3}{12}$$

$$\therefore \quad \boxed{I_{YY} = 12 \times 10^6 \text{ mm}^4}$$

Important Multiple Choice Questions

Q.1 *The unit of radius of gyration is _____.*

a) mm *b) mm^2* *c) mm^3* *d) mm^4*

VTU : Feb.-09, June-12,13, Jan.-13

(Ans. : a)

Q.2 *The moment of inertia of an area about an axis which is in a plane perpendicular to the area is called _____ .*

a) radius of gyration *b) polar moment of inertia*

c) second moment of area *d) none of the above*

VTU : Feb.-09, June-13

(Ans. : b)

Q.3 *The moment of inertia of a circle with 'D' as its diameter about its centroidal axis is _____.*

a) $\frac{\pi D^2}{32}$ *b)* $\frac{\pi D^2}{64}$ *c)* $\frac{\pi D^4}{32}$ *d)* $\frac{\pi D^4}{64}$

VTU : Feb.-09, Dec.-11,13, June-13

(Ans. : d)

Q.4 *The moment of inertia of a square of side 'b' about an axis through its centroid is _____.*

a) $\frac{b^4}{12}$ *b)* $\frac{b^4}{8}$ *c)* $\frac{b^4}{36}$ *d)* $\frac{b^3}{12}$

VTU : Feb.-09, 10, June-12,13, Jan.-13

(Ans. : a)

Q.5 *Moment of inertia is a _____.*

a) first moment of area *b) second moment of area*

c) third moment of area *d) none of the above*

VTU : Aug.-09, Dec.-11, Jan.-13

(Ans. : b)

Q.6 *M.I. of circular section about centroidal axis is _____.*

a) $\frac{\pi D^4}{48}$ *b)* $\frac{\pi D^4}{32}$ *c)* $\frac{\pi D^4}{64}$ *d)* $\frac{\pi D^4}{128}$

VTU : Aug.-09, Jan.-13

(Ans. : c)

Q.7 *M.I. of triangular section about base having base 'b' and height 'h' is _____.*

a) $\frac{bh^3}{36}$ *b)* $\frac{bh^3}{12}$ *c)* $\frac{bh^3}{64}$ *d) none*

VTU : Aug.-09, Feb.-10

(Ans. : b)

Q.8 *M.I. of hollow circular section whose external diameter is 8 mm and internal diameter 4 mm about centroidal axis is _____.*

a) $437.5\ mm^4$ *b)* $337.5\ mm^4$ *c)* $237.5\ mm^4$ *d)* $188.4\ mm^4$

VTU : Aug.-09

(Ans. : d)

Q.9 *The unit of moment of inertia of an area is _____.*

a) m^3 *b)* m^2 *c)* m^4 *d)* N/m^2

VTU : Feb.-10

(Ans. : c)

Q.10 *The polar moment of inertia of a circular section of diameter 'D' about its centroidal axis is _____.*

a) $\frac{\pi D^4}{64}$ *b)* $\frac{\pi D^4}{32}$ *c)* $\frac{\pi D^3}{32}$ *d)* $\frac{\pi D^4}{16}$

VTU : Feb.-10

(Ans. : b)

Q.11 *If I_G is M.I. of a rectangle about its centroidal axis and I_{AB} is M.I. about its base, then _____.*

a) $I_G > I_{AB}$ *b)* $I_G < I_{AB}$ *c)* $I_G = I_{AB}$ *d) none of the above*

VTU : Feb.-11

(Ans. : a)

Q.12 *If K_{XX} = 3 mm and K_{YY} = 4 mm for some area, then K_{ZZ} = _____.*

a) 3 mm *b) 4 mm* *c) 5 mm* *d) 6 mm*

(Ans. : c)

Q.13 *M.I. of a circular area of radius R about a tangent in its plane is _____.*

a) $\frac{\pi R^4}{4}$ *b)* $\frac{3\pi R^4}{4}$ *c)* $\frac{\pi R^4}{2}$ *d)* $\frac{3\pi R^4}{2}$ **(Ans. : c)**

Q.14 *M.I. of a circular area of radius R about a tangent perpendicular to its plane is _____.*

a) $\frac{\pi R^4}{4}$ *b)* $\frac{3\pi R^4}{4}$ *c)* $\frac{\pi R^4}{2}$ *d)* $\frac{3\pi R^4}{2}$ **(Ans. : b)**

Q.15 *The cross section area of a hollow cylinder has an internal diameter of 50 mm and a thickness of 5 mm. The M.I. of the cross section about its centroidal axis is _____.*

a) 3.294×10^5 mm^4 *b)* 1.424×10^5 mm^4

c) 1.647×10^5 mm^4 *d)* 2.848×10^5 mm^4 **(Ans. : a)**

Q.16 *M.I. of triangular section of base b and height h about its centroidal axis paralel to its base is _____.*

a) $\frac{bh^3}{36}$ *b)* $\frac{bh^3}{12}$ *c)* $\frac{bh^3}{64}$ *d) none of the above*

(Ans. : a)

Q.17 *M.I. of an equilateral triangle of side 'b' about its base is _____.*

a) $\frac{b^4}{12}$ *b)* $\frac{b^4}{36}$ *c)* $\frac{\sqrt{2}b^4}{32}$ *d)* $\frac{\sqrt{3}b^4}{32}$ **(Ans. : d)**

Q.18 *M.I. of a triangle of base b and height h about an axis passing through its apex and parallel to its base is _____.*

a) $\frac{bh^3}{12}$ *b)* $\frac{bh^3}{36}$ *c)* $\frac{bh^3}{4}$ *d)* $\frac{dh^3}{3}$ **(Ans. : c)**

Q.19 *If I_{AB} is the moment of inertia of an area about an axis AB in its plane, I_G is M.I. of the area about its centroidal axis and d is the distance between the two parallel axes, then* _____ .

a) $I_G = I_{AB} + Ad^2$ *b)* $I_G = I_{AB} - Ad^2$ *c)* $I_G = I_{AB} + Ad$ *d)* $I_G = I_{AB} - Ad$

(Ans. : b)

Q.20 *Which of the following equations relating the three radii of gyration about X, Y and Z axes is correct where Z axis is perpendicular to the plane of the area ?*

a) $K_{ZZ} = K_{XX} + K_{YY}$ *b)* $K_{XX} = K_{YY} + K_{ZZ}$

c) $K_{ZZ}^2 = K_{XX}^2 + K_{YY}^2$ *d) None of the above*

VTU : June-12

(Ans. : c)

Q.21 *The radius of gyration of a circular area of radius R about a centroidal axis in its plane is* _____.

a) R *b)* $\frac{R}{2}$ *c)* $\frac{R}{4}$ *d)* $\frac{3R}{2}$

(Ans. : b)

Q.22 *The radius of gyration of a square of side 'b' about a centroidal axis in its plane is* _____.

a) $\frac{b}{2\sqrt{3}}$ *b)* $\frac{b}{3\sqrt{2}}$ *c)* $\frac{b^2}{2\sqrt{3}}$ *d)* $\frac{b^2}{3\sqrt{2}}$

(Ans. : a)

Q.23 *The moment of inertia about centroidal X-axis is given by* I_{XX} = _____.

a) $\int x^2\,dA$ *b)* $\int y^2 dA$ *c)* $\int x\,dA$ *d)* $\int y\,dA$

(Ans. : b)

Q.24 *The moment of inertia about centroidal Y-axis is given by* I_{YY} = _____.

a) $\int x^2\,dA$ *b)* $\int y^2\,dA$ *c)* $\int x\,dA$ *d)* $\int y\,dA$

(Ans. : a)

Q.25 *The radius of gyration of a triangle about its base is* _____.

a) $\frac{b}{2\sqrt{3}}$ *b)* $\frac{b}{3\sqrt{2}}$ *c)* $\frac{h}{2\sqrt{3}}$ *d)* $\frac{h}{3\sqrt{2}}$

(Ans. : d)

Q.26 *The moment of inertia of a rectangular lamina of sides 'b' and 'h' about its neutral axis parallel to the sides 'b' is given by* _____ .

VTU : Aug.-10

a) $\frac{bh^3}{12}$ *b)* $\frac{bh^3}{36}$ *c)* $\frac{hb^3}{12}$ *d)* $\frac{hb^3}{36}$

(Ans. : a)

Q.27 *The unit of radius of gyration of an area is* _____ .

VTU : Aug.-10

a) N/m *b)* N/m^2 *c)* m^3 *d) m*

(Ans. : d)

Q.28 *The value of moment of inertia depends upon _____ .* **VTU : Aug.-10, 11**

a) type of material *b) weight of material*

c) density of material *d) cross-sectional dimensions* **(Ans. : d)**

Q.29 *Moment of inertia is a _____ .* **VTU : Aug.-10**

a) first order term *b) second order term*

c) fourth order term *d) third order term* **(Ans. : c)**

Q.30 *The moment of inertia for triangle ($b \times h$) about its base is _____.* **VTU : Feb.-11**

a) $bh^3/2$ *b)* $bh^3/4$ *c)* $bh^3/8$ *d)* $bh^3/12$ **(Ans. : d)**

Q.31 *The radius of gyration of a circular area of radius "r" is _____.* **VTU : Feb.-11**

a) r/2 *b) r/4* *c) 2r/3* *d) 3r/4* **(Ans. : a)**

Q.32 *The unit of moment of inertia of an area is _____ .* **VTU : Feb.-11**

a) kg.m *b)* $kg.m^2$ *c)* $kg.m^4$ *d)* m^4 **(Ans. : d)**

Q.33 *The moment of inertia of a body is _____ .* **VTU : Aug.-11**

a) moment of its inertia

b) the rotational moment acting on the body

c) the rotational analogue of mass

d) the inertial moment acting on the body **(Ans. : c)**

Q.34 *The second moment of a plane area about any axis as compared to its second moment about the neutral axis _____ .* **VTU : Aug.-11**

a) is always more *b) is always less* *c) is equal* *d) not equal* **(Ans. : a)**

Q.35 *Moment of inertia of a square of side 'a' about an axis through its centre of gravity is _____ .*

a) $\frac{a^4}{4}$ *b)* $\frac{a^4}{8}$ *c)* $\frac{a^4}{12}$ *d)* $\frac{a^4}{36}$ **(Ans. : c)**

Q.36 *Polar moment of inertia of a plane area is _____ .* **VTU : Dec.-11**

a) $I_{xx} \times I_{yy}$ *b)* $I_{xx} + I_{yy}$ *c)* I_{xx}/I_{yy} *d) None of these* **(Ans. : b)**

Q.37 *The unit of moment of inertia of an area is _____ .* **VTU : Dec.-11**

a) m^2 *b)* m *c)* m^4 *d)* m^3 **(Ans. : c)**

Q.38 *Moment of inertia of a triangle of base 'b' and height 'h' about its base is _____ .*

a) $\frac{bh^3}{36}$ *b)* $\frac{bh^4}{36}$ *c)* $\frac{hb^3}{12}$ *d)* $\frac{bh^3}{12}$

VTU : June-11

(Ans. : d)

Q.39 *Unit of second moment of area is _____.* **VTU : Dec.-13**

a)m *a)* m^2 *b)* m^4 *c)* m^5 **(Ans. : c)**

Q.40 *Unit of radius of gyration is _____.* **VTU : Dec.-13**

a) m *b)* m^2 *c)* m^3 *d)* m^4 **(Ans. : a)**

Q.41 *Polar moment of inertia is _____.* **VTU : Dec.-13**

a) $I_{xx}+I_{yy}$ *b)* $I_{xx}+I_{zz}$ *c)* $I_{yy}+I_{zz}$ *d) none of these*

(Ans. : a)

❑❑❑

10 Kinematics

Chapter at a Glance

Rectilinear Motion

- Average velocity $v_{av} = \frac{\Delta x}{\Delta t}$
- Instantaneous velocity $v = \frac{dx}{dt}$
- Average acceleration $a_{av} = \frac{\Delta v}{\Delta t}$
- Instantaneous acceleration $a = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d^2x}{dt^2}$
- Equations for constant acceleration :

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s_n - s_{n-1} = u + \frac{1}{2}a(2n-1)$$

- For motion under gravity, use $a = -g$ in equations for constant acceleration.

Curvilinear Motion

- Average velocity $\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t}$
- Instantaneous velocity $\vec{v} = \frac{d\vec{r}}{dt}$
- Average acceleration $\vec{a}_{av} = \frac{\Delta \vec{v}}{\Delta t}$
- Instantaneous acceleration $\vec{a} = \frac{d\vec{v}}{dt}$

- In Rectangular components,

$$\vec{r} = x\,\hat{i} + y\,\hat{j}$$

$$\vec{v} = v_x\hat{i} + v_y\hat{j} \qquad \text{where} \quad v_x = \frac{dx}{dt} \text{ and } v_y = \frac{dy}{dt}$$

$$\vec{a} = a_x\,\hat{i} + a_y\,\hat{j} \qquad \text{where} \quad a_x = \frac{dv_x}{dt}; \quad a_y = \frac{dv_y}{dt}$$

$$a = \sqrt{a_x^2 + a_y^2}, \quad \theta_a = \tan^{-1}\left(\frac{|a_y|}{|a_x|}\right)$$

Projectile Motion

- In x - direction : $v_x = u\cos\alpha$ and $x = (u\cos\alpha)\cdot t$
- In y - direction : $v_y = u\sin\alpha - gt$, $y = (u\sin\alpha)t - \frac{1}{2}gt^2$ and $v_y^2 = u^2\sin^2\alpha - 2gy$
- Equation of path : $y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$
- Range on horizontal plane : $R = \frac{u^2\sin 2\alpha}{g}$
- Time of flight on horizontal plane : $T = \frac{2u\sin\alpha}{g}$
- Maximum height above the initial point : $H = \frac{u^2\sin^2\alpha}{2g}$
- Range on inclined plane : $R = \frac{2u^2\cos\alpha\sin(\alpha-\beta)}{g\cos^2\beta}$

Relative Motion

- Velocity of A with respect to B : $\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$
- Acceleration of A with respect to B : $\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$

Kinetics of Rectilinear Motion

- $\sum F_x == ma_x$
- $\sum F_y \quad ma_y$

Motion of Vehicles on Banked Curved Roads

- If θ is the angle of banking,

$$\tan\theta = \frac{v^2}{\rho g}$$

- Superelevation $h = W \sin\theta$

Method for Solving Problems on Variable Acceleration

1) Use basic definitions for velocity and acceleration.

2) Either differentiate or integrate the functions.

Method for Solving Problems on Constant Acceleration

- Use $v = u + at$

$$s = ut + \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

$$s_n - s_{n-1} = u + \frac{1}{2}a(2n-1)$$

Method for Solving Problems on Motion Under Gravity

- For motion under gravity, use $a = -g$ in equations for constant acceleration.

Method for Solving Problems on Projectile Motion

- If initial and final points are at the same level, use $R = \frac{u^2 \sin 2\alpha}{g}$, $T = \frac{2u \sin\alpha}{g}$, $H = \frac{u^2 \sin^2\alpha}{2g}$

- If initial and final points are not at the same level, use equation of path

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

or the basic equations in x and y directions.

Method for Solving Problems on Relative Motion

i) To calculate relative velocity, write velocity vectors $\vec{v}_A$ and $\vec{v}_B$ in terms of unit vectors $\hat{i}$ and $\hat{j}$. Then use

$$\vec{v}_{A/B} = \vec{v}_A - \vec{v}_B$$

To calculate relative acceleration, write acceleration vectors $\vec{a}_A$ and $\vec{a}_B$ in terms of $\hat{i}$ and $\hat{j}$. Then use

$$\vec{a}_{A/B} = \vec{a}_A - \vec{a}_B$$

ii) To find shortest distance

a) Find relative velocity $\vec{v}_{A/B}$ or $\vec{v}_{B/A}$ in magnitude and direction.

b) Plot initial positions of A and B.

c) If $\vec{v}_{A/B}$ has been calculated in (a), then B is the observer and hence is treated stationary. We plot the motion of A as observed by B which is represented by $\vec{v}_{A/B}$. Plot the direction of A as observed by B in proper quadrant. (It is the direction of $\vec{v}_{A/B}$).

d) Draw perpendicular from B on the direction of $\vec{v}_{A/B}$ and calculate this perpendicular distance which is the shortest distance using basic trigonometry.

iii) If A has to meet B or A has to hit B, the shortest distance between A and B must be zero. Then the relative velocity will be along the line joining A and B.

Important Solved Examples

Example 10.1 *The motion of a particle starting from rest is defined by $a = 10t - t^2$, where a is in m/s^2 and t is in seconds. Find the displacement before it starts in reverse direction of motion and velocity when acceleration changes its direction.* **PU : May-08**

Solution : $a = 10t - t^2 = \frac{dv}{dt}$

$$\int_0^v dv = \int_0^t (10t - t^2)\, dt$$

$$[v]_0^v = \left[5t^2 - \frac{t^3}{3}\right]_0^t$$

$$\therefore \quad v = 5t^2 - \frac{t^3}{3} \qquad \ldots (1)$$

$$v = \frac{dx}{dt}$$

$$\therefore \quad \frac{dx}{dt} = 5t^2 - \frac{t^3}{3}$$

$$\int_0^x dx = \int_0^t \left(5t^2 - \frac{t^3}{3}\right) dt$$

$$\therefore \quad x = \frac{5t^3}{3} - \frac{t^4}{12} \qquad \text{... (2)}$$

When particle reverses direction, $v = 0$

$$\therefore \quad 5t^2 - \frac{t^3}{3} = 0$$

$$5 - \frac{t}{3} = 0$$

$$\therefore \quad t = 15 \text{ s}$$

At $t = 15$ s , displacement is,

$$x = \frac{5 \times 15^3}{3} - \frac{15^4}{12}$$

$$\therefore \quad \boxed{x = 1406.25 \text{ m}}$$

When acceleration changes direction, $a = 0$

$$\therefore \quad 10t - t^2 = 0$$

$$\therefore \quad t = 10 \text{ s}$$

At $t = 10$ s,

$$v = 5 \times 10^2 - \frac{10^3}{3}$$

$$\therefore \quad \boxed{v = 166.67 \text{ m/s}}$$

Example 10.2 *The motion of the particle is defined by the relation $x = t^2 - (t-3)^3 m$ where 'x' and 't' are in meters and seconds respectively. Determine :*

a) The time when velocity is maximum. b) The position and maximum velocity.

c) The distance travelled at t = 12 seconds. **P U : Dec.-11**

Solution : $x = t^2 - (t-3)^3$

$$v = \frac{dx}{dt} = 2t - 3(t-3)^2$$

$$a = \frac{dv}{dt} = 2-6(t-3)$$

a) For maximum velocity, $a = 0$

$\therefore \quad 2-6(t-3) = 0$

$$2-6t+18 = 0$$

$$6t = 16$$

$\therefore \quad \boxed{t = 3.33 \text{ s}}$

b) $x_{3.33} = 3.33^2 - (3.33-3)^3$

$\therefore \quad \boxed{x_{2.667} = 11.05 \text{ m}}$

$$v_{max} = 2 \times 3.33 - 3(3.33-3)^2$$

$\therefore \quad \boxed{v_{max} = 6.33 \text{ m/s}}$

c) To find distance travelled, put $v = 0$

$\therefore \quad 2t-3(t-3)^2 = 0$

$$2t-3(t^2-6t+9) = 0$$

$$3t^2-20t+27 = 0$$

$\therefore \quad t = 1.88 \text{ s}, 4.786 \text{ s}.$

$$x_0 = 0-(0-3)^3 = 27 \text{ m}$$

$$x_{1.88} = 1.88^2-(1.88-3)^3 = 4.94 \text{ m}$$

$$x_{4.786} = 4.786^2-(4.786-3)^3 = 17.21 \text{ m}$$

$$x_{12} = 12^2-(12-3)^3 = -585 \text{ m}$$

Distance travelled $= |x_{1.88}-x_0| + |x_{4.786}-x_{1.88}| + |x_{12}-x_{4.786}|$

$$= |4.94-27| + |17.21-4.94| + |-585-17.21|$$

$$= 22.06 + 12.27 + 602.21$$

$\therefore \quad \boxed{\text{Distance travelled} = 636.54 \text{ m}}$

Example 10.3 *A particle starts with an initial velocity of 2.5 m/s and uniformly accelerates at the rate 0.5 m/s^2. Determine the displacement in 2 s, time required to attain the velocity of 7.5 m/s and the distance travelled when it attain a velocity of 7.5 m/s.*

PU : Dec.-12

Solution : $u = 2.5$ m/s, $a = 0.5$ m/s^2

$$s = ut + \frac{1}{2}at^2$$

At $t = 2$ s, $\quad s = 2.5\times 2 + \frac{1}{2}\times 0.5\times 2^2$

$\therefore \quad \boxed{s = 6 \text{ m}}$

$$v = u + at$$

For $\quad v = 7.5$ m/s,

$$7.5 = 2.5 + 0.5\times t$$

$\therefore \quad \boxed{t = 10 \text{ s}}$

$$v^2 = u^2 + 2as$$

$$7.5^2 = 2.5^2 + 2(0.5)(s)$$

$\therefore \quad \boxed{s = 50 \text{ m}}$

Example 10.4 *A base ball is thrown downward from a 15 m tower with an initial speed of 5 m/s. Determine the speed at which it hits the ground and the time of travel.*

PU : May-11, 13, Marks 6

Solution : $u = 5$ m/s, $\quad s = -15$ m,

$$a = -9.81 \text{ m/s}^2$$

$$v^2 = u^2 + 2as$$

$\therefore \quad v^2 = (-5)^2 + 2(-9.81)(-15)$

$\therefore \quad \boxed{v = 17.87 \text{ m/s} \downarrow}$

$$v = u + at$$

$$-17.87 = (-5) + (-9.81)t$$

$\therefore \quad \boxed{t = 1.312 \text{ s}}$

Example 10.5 *The Y-co-ordinate of a particle moving along a curve is $y = t^3 - 6t + 3$ where y is in metres and t in seconds. Its acceleration in x-direction is given by $a_x = 4t + 3$ m/s^2. If velocity of the particle in x direction is 2 m/s when $t = 0$, calculate the magnitude and direction of velocity and acceleration of the particle when $t = 1$ s.*

Solution : $y = t^3 - 6t + 3$

$$v_y = \frac{dy}{dt} = 3t^2 - 6$$

$$a_y = \frac{d\,v_y}{dt} = 6t$$

$$a_x = 4t + 3$$

$$\therefore \quad \frac{dv_x}{dt} = 4t + 3$$

$$v_x = \frac{4t^2}{2} + 3t + C_1$$

$$\therefore \quad v_x = 2t^2 + 3t + C_1$$

At $t = 0$, $v_x = 2$ m/s

$$\therefore \quad 2 = C_1$$

$$\therefore \quad v_x = 2t^2 + 3t + 2$$

At $t = 1$ s,

$$v_x = 2 + 3 + 2 = 7 \text{ m/s}$$

$$v_y = 3 - 6 = -3 \text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{7^2 + 3^2} = 7.62 \text{ m/s}$$

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{3}{7} = 23.2°$$

$$a_x = 4 + 3 = 7 \text{ m/s}^2 \; ; \quad a_y = 6 \times 1 = 6 \text{ m/s}^2$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{7^2 + 6^2} = 9.22 \text{ m/s}^2$$

$$\theta_a = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\frac{6}{7} = 40.6°$$

Example 10.6 *A cricket ball thrown from a height of 1.8 above ground level at an angle of 30° with the horizontal with a velocity of 12 m/s is caught by a fielder at a height of 0.6 m above the ground. Determine the distance between the two players.*

Solution : The equation of path is,

$$y = x \tan\alpha - \frac{g x^2}{2u^2 \cos^2 \alpha}$$

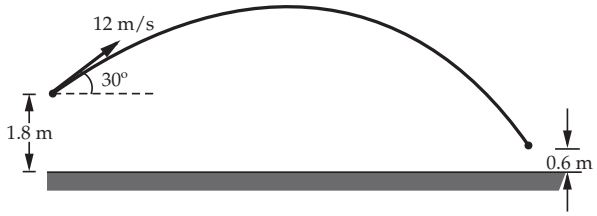


Fig. 10.1

Here $y = -1.2, \quad x = ?$

$g = 9.81, \quad \alpha = 30°, \quad u = 12$ m/s

$$\therefore \quad -1.2 = x \tan 30 - \frac{9.81 x^2}{2 \times 12^2 \cos^2 30}$$

$$\therefore \quad \frac{9.81 x^2}{2 \times 144 \cos^2 30} - \tan 30 \times x - 1.2 = 0$$

$$\therefore \quad x = 14.53 \text{ m}, \; -1.82 \text{ m}$$

x cannot be negative

$$x = 14.53 \text{ m}$$

Example 10.7 *Two ships 'A' and 'B' are at a given instant 4 km away from each other and both are on south-east line. Ship 'A' is travelling at 8 kmph due east and ship 'B' is travelling at 12 kmph due north. Determine -*

i) Velocity of 'B' with respect to 'A'.

ii) The shortest distance between the two ships.

iii) Time to get the shortest distance.

PU : Dec.-09

Solution : The velocities of '*A*' and '*B*' are shown in Fig. 10.2 (a) and (b).

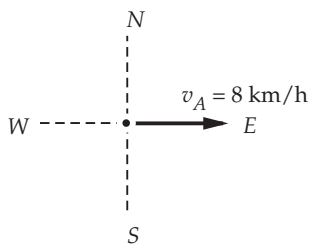


Fig. 10.2 (a)

$$\therefore \quad \vec{v}_A = 8\,\hat{i}$$

$$\vec{v}_B = 12\,\hat{j}$$

$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A = 12\,\hat{j} - 8\,\hat{i}$$

$$\therefore \quad \vec{v}_{B/A} = -8\,\hat{i} + 12\,\hat{j}$$

$$\vec{v}_{B/A} = \sqrt{8^2 + 12^2}$$

$\therefore$ $v_{B/A} = 14.422$ km/h

$$\theta = \tan^{-1}\left(\frac{12}{8}\right)$$

$\therefore$ $\theta = 56.31^\circ$

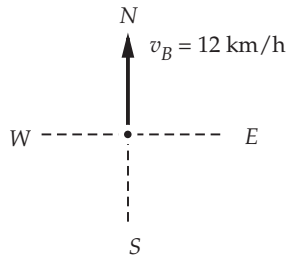


Fig. 10.2 (b)

The initial positions of A and B, and $v_{B/A}$ are shown in Fig. 10.2. (c).

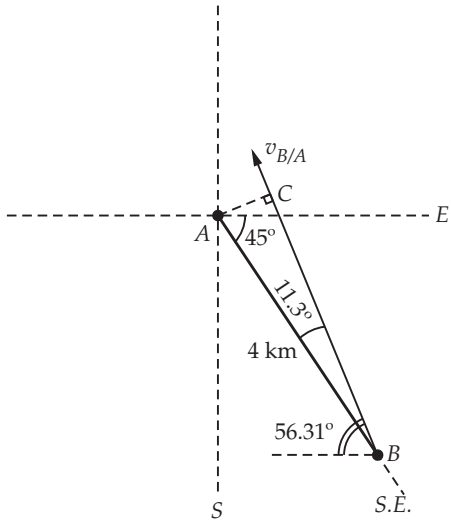


Fig. 10.2 (c)

The shortest distance is AC.

$$AC = 4 \sin 11.31$$

$\therefore$ $AC = 0.784$ km

Time to get the shortest distance is

$$t = \frac{BC}{v_{B/A}} = \frac{4 \cos 11.31}{14.422}$$

$\therefore$ $t = 0.272$ hrs

or

$\therefore$ $t = 16$ min 19 sec

Example 10.8 *Determine the apparent weight of a 70 kg man in an elevator when the acceleration of elevator is a) Zero, b) 2 m/s^2 upwards and c) 2 m/s^2 downwards.*

Solution : Apparent weight is the normal reaction. When acceleration of elevator is zero, there is no inertia force due to motion of elevator on the man. The *F.B.D.* is shown in Fig. 10.3 (a).

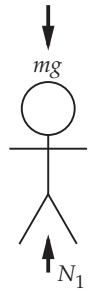


Fig. 10.3 (a)

$$\sum F_y = 0 :$$

$$N_1 - mg = 0$$

$$N_1 = 70 \times 9.81$$

$$\therefore \quad \boxed{N_1 = 686.7 \text{ N}}$$

When lift accelerates upwards, the inertia force *ma* is downwards as shown in Fig. 10.3 (b).

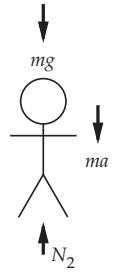


Fig. 10.3 (b)

$$\sum F_y = 0 :$$

$$N_2 - mg - ma = 0$$

$$N_2 = 70 \times 9.81 + 70 \times 2$$

$$\therefore \quad \boxed{N_2 = 826.7 \text{ N}}$$

When lift accelerates downwards, the inertia force *ma* is upwards as shown in Fig. 10.3 (c).

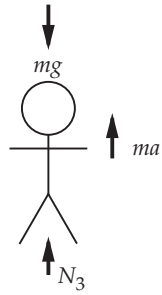


Fig. 10.3 (c)

$$\sum F_y = 0 :$$

$$N_3 - mg + ma = 0$$

$$N_3 = 70 \times 9.81 - 70 (2)$$

$$\therefore \quad \boxed{N_3 = 54.67 \text{ N}}$$

Important Multiple Choice Questions

Q.1 *Object decelerates when* _____.

a) acceleration is negative *b) acceleration is positive*

c) acceleration is directed opposite to velocity *d) none of the above*

(Ans. : c)

Q.2 *At the point where particle changes direction in rectilinear motion, the velocity is* _____.

a) zero *b) maximum* *c) minimum* *d) none of the above*

(Ans. : a)

Q.3 *Negative value of acceleration is _____.*

a) always retardation *b) always acceleration*

c) retardation if v is positive and acceleration if v is negative

d) none of the above

(Ans. : c)

Q.4 *If $a = 0$, then _____.*

a) $v = 0$ *b) v = constant* *c) v is either maximum or minimum*

d) any of the above

(Ans. : d)

Q.5 *If a particle has variable acceleration and its acceleration becomes zero at a particular instant of time, then _____.*

a) $v = 0$ *b) v = constant* *c) v is either maximum or minimum*

d) any of the above

(Ans. : c)

Q.6 *If position of a particle varies linearly with time, then _____.*

a) $x = 0$ *b) $v = 0$* *c) $a = 0$* *d) $t = 0$*

(Ans. : c)

Q.7 *A particle starts from rest with an acceleration $a = 3\ t\ \mathrm{m/s^2}$. To find the time at which $v = 5\ \mathrm{m/s}$, we can use _____.*

a) $v = u + at$ *b) $s = ut + \frac{1}{2}at^2$* *c) $v^2 = u^2 + 2as$* *d) none of the above*

(Ans. : d)

Q.8 *If the distance covered by a particle is zero, its displacement _____.*

a) must be zero *b) cannot be zero* *c) is negative*

d) may or may not be zero

(Ans. : a)

Q.9 *The displacement of a particle is given by $x = a + bt + ct^2 + dt^3$. The initial velocity of the particle is _____.*

a) a *b) b* *c) 2 c* *d) 3 d*

(Ans. : b)

Q.10 *Motion under gravity is motion with _____.*

a) constant velocity *b) increasing acceleration*

c) decreasing acceleration *d) constant acceleration*

(Ans. : d)

Q.11 *Ball A is thrown upward from the top of a tower with speed v_0. At the same instant another ball B is thrown down with speed v_0. If v_A and v_B are velocities of A and B just before striking the ground, then _____.*

a) $v_A > v_B$ *b)* $v_A < v_B$

c) v_A *and* v_B *depend on masses of A and B* *d)* $v_A = v_B$

(Ans. : d)

Q.12 *An object is projected upward with velocity* v_0. *The maximum height reached by it is* _____.

a) $\sqrt{v_0 g}$ *b)* $\frac{v_0^2}{2g}$ *c)* $2v_0 g$ *d)* $\frac{2g}{v_0^2}$

(Ans. : b)

Q.13 *A ball is projected up with velocity* v_0. *At maximum height reached by it, its acceleration is* _____.

a) 0 *b)* 9.81 $m/s^2 \uparrow$ *c)* 9.81 $m/s^2 \downarrow$ *d) none of the above*

(Ans. : c)

Q.14 $\vec{v}_{A/B}$ = _____.

a) $\vec{v}_A + \vec{v}_B$ *b)* $\vec{v}_A - \vec{v}_B$ *c)* $v_A - v_B$ *d)* $\vec{v}_B - \vec{v}_A$

(Ans. : b)

Q.15 $\vec{v}_B$ = _____.

a) $\vec{v}_A + \vec{v}_{A/B}$ *b)* $\vec{v}_A - \vec{v}_{A/B}$ *c)* $\vec{v}_{A/B} - \vec{v}_A$ *d)* $-\vec{v}_A - \vec{v}_{A/B}$

(Ans. : b)

Q.16 *When observed from a stationary train, raindrops appear to fall vertically. If the train starts moving towards east with constant velocity the drops will appear to fall* _____.

a) vertically *b) at an angle towards west*

c) at an angle towards east *d) at an angle towards north*

(Ans. : b)

Q.17 *A body moving in a circular path with constant speed has* _____ .

a) constant velocity *b) constant acceleration*

c) constant displacement *d) none of the above*

(Ans. : d)

Q.18 *The path of a projectile is* _____ .

a) straight line *b) parabola* *c) circle* *d) hyperbola*

(Ans. : b)

Q.19 *For a projectile,* _____ *is constant.*

a) velocity *b) displacement* *c) speed* *d) acceleration*

(Ans. : d)

Q.20 *The horizontal range of a projectile is* _____ .

a) $\frac{u^2 \sin^2 \alpha}{2g}$ b) $\frac{u^2 \sin 2\alpha}{2g}$ c) $\frac{u^2 \sin 2\alpha}{g}$ d) $\frac{u^2 \sin^2 \alpha}{g}$

(Ans. : c)

Q.21 *The horizontal range is maximum when the angle of projection α is _____ .*

a) 0^o b) 30^o c) 45^o d) 60^o

(Ans. : c)

Q.22 *The maximum height reached by a projectile above the point from which it was projected is _____ .*

a) $\frac{u^2 \sin^2 \alpha}{g}$ b) $\frac{u^2 \sin^2 \alpha}{2g}$ c) $\frac{u^2 \sin 2\alpha}{g}$ d) $\frac{u^2 \sin 2\alpha}{2g}$

(Ans. : b)

Q.23 *The time of flight for projectile motion on horizontal plane is _____ .*

a) $\frac{2u \sin \alpha}{g}$ b) $\frac{u \sin \alpha}{g}$ c) $\frac{u^2 \sin^2 \alpha}{2g}$ d) $\frac{u^2 \sin 2\alpha}{g}$

(Ans. : a)

Q.24 *The time required by a projectile to reach maximum height is _____ .*

a) $\frac{2u \sin \alpha}{g}$ b) $\frac{u \sin \alpha}{g}$ c) $\frac{u^2 \sin^2 \alpha}{2g}$ d) $\frac{u^2 \sin 2\alpha}{g}$

(Ans. : b)

Q.25 *The equation of path for projectile motion is _____ .*

a) $y = x \tan \alpha - \frac{x^2}{2ug \cos^2 \alpha}$ b) $y = x \tan \alpha - \frac{gx^2}{2u \cos^2 \alpha}$

c) $y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}$ d) $y = x \tan \alpha - \frac{gx^2}{2u \cos \alpha}$

(Ans. : c)

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