As per Revised Syllabus of

VISVESVARAYA TECHNOLOGICAL UNIVERSITY

ENGINEERING PHYSICS

(One day Revision Book)

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1 Modern Physics

Chapter at a Glance

- A body which absorbs all the radiation incident on it without reflection is known as a black body.
- **Stefan - Boltzmann law :** The total energy radiated by a black body per unit area per unit time is proportional to the fourth power of its absolute temperature i.e.,

$$\therefore \qquad E = \sigma T^4$$

where σ is the Stefan's constant.

- **Wien's displacement laws :**

$$\lambda T = \text{Constant}$$

and $$E_\lambda T^{-5} = \text{Constant}$$

- where λ is the wavelength with greatest energy density, T = Absolute temperature and E_λ is the energy radiated per unit area per unit time.
- Wien's law fails to predict the observations for long wavelengths and very high temperatures.
- All photons associated with light of frequency ν have the same energy given by,

$$E = h\nu$$

where $h = 6.626 \times 10^{-34}$ J.s. *is the Planck's constant.*

- Rayleigh-Jeans formula is in good agreement with observations at long wavelengths but fails at small wavelengths.
- Planck's law is able to account for the complete black body spectrum.
- The increase in wavelength of X-rays due to scattering is known as **Compton effect** and change in wavelength is known as **Compton shift**.
- The Compton shift is given by

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\phi)$$

- **De-Broglie Hypothesis :** Every moving particle is associated with a wave, called the matter wave or De-Broglie wave, whose wavelength (λ) is given by,

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

where h = Planck's constant, m = Mass of the particle and v = Particle velocity.

- For a free particle with kinetic energy (E),

$$\lambda = \frac{h}{\sqrt{2mE}}$$

- For an electron is accelerated through a potential difference of 'V' volts,

$$\lambda = \frac{12.27}{\sqrt{V}} \text{ A}^{\circ}$$

- Phase velocity (also known as wave velocity) is the velocity with which a particular phase of the wave propagates in a medium.
- Velocity of De-Broglie waves is always greater than the velocity of light.
- The De-Broglie wave group associated with moving particle travels with the same velocity as the particle.
- Relation between Phase Velocity and Group Velocity : $v_g = v_p - \lambda \frac{dv_p}{d\lambda}$

Important Theory Questions and Answers

Q.1 *What is Planck's radiation law ? Show how Wien's law and Rayleigh-Jean's law can be derived from it.* **VTU : Aug.-08, Feb.-10, Marks 6**

Ans. : Max Planck assumed that the radiation is emitted in packets or bundles of energy called quanta or photons instead of continuous waves. All photons associated with light of frequency ν have the same energy given by,

$$E = h\nu$$

where $h = 6.626 \times 10^{-34}$ J.s. is the Planck's constant.

Planck considered the electromagnetic waves in a cavity as a photon gas which follows the Bose-Einstein distribution function applicable to identical, indistinguishable particles with integer spins. The energy density of photons in a cavity was obtained as

$$\therefore \quad E_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} d\lambda \quad \ldots (1)$$

Wien's displacement law from Planck's law

Planck's radiation law gives the energy density in wavelength region λ to $\lambda + d\lambda$ as,

$$E_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5} \times \left(\frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \right) d\lambda \qquad \ldots (2)$$

For small temperatures or small wavelength, λT is small.

$$\therefore \qquad e^{\frac{hc}{\lambda kT}} >> 1$$

Neglecting 1 in comparison with $e^{\frac{hc}{\lambda kT}}$ in equation (2)

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}}} d\lambda$$

$$\therefore \qquad E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{\frac{-hc}{\lambda kT}} d\lambda$$

The above equation, which is valid for small values of λ and T is the Wien's law. To find the wavelength at which maximum energy is radiated, the mathematical condition is,

$$\frac{dE_\lambda}{d\lambda} = 0$$

Using this condition, we get

$$\lambda_{max} T = 2.898 \times 10^{-3} \text{ mK} \qquad \ldots (3)$$

The above equation predicts that the wavelength at which maximum amount of radiation is emitted shifts towards shorter wavelengths i.e., higher frequencies as temperature is increased.

Rayleigh Jeans law from Planck's law

For large values of λT (i.e. for large T and/or λ), $\frac{hc}{\lambda kT}$ becomes very small.

$$\therefore \qquad e^{\frac{hc}{\lambda kT}} = 1 + \frac{hc}{\lambda kT} + \frac{1}{2!}\left(\frac{hc}{\lambda kT} \right)^2 + \cdots$$

Neglecting higher order term,

$$e^{\frac{hc}{\lambda kT}} = 1 + \frac{hc}{\lambda kT}$$

Substituting in Planck's radiation formula,

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{1}{1+\frac{hc}{\lambda kT}-1}\right]d\lambda$$

$$\therefore \quad E_\lambda d\lambda = \frac{8\pi kT}{\lambda^4}d\lambda \quad \ldots (4)$$

Which is Rayleigh Jeans law.

Q.2 *Define phase velocity and group velocity. Show that group velocity is same as particle velocity.* **VTU : Feb.-09, Aug.-10, Marks 8**

Ans. : **Phase velocity** (also known as wave velocity) is the velocity with which a particular phase of the wave propagates in a medium.

Mathematically, a wave group can be described interms of a superposition of individual waves of different wavelengths. The interference of these waves gives variation in amplitude defining the shape of the wave group. The velocity of this group of waves is known as **group velocity**.

If the two waves have angular velocities differing by $d\omega$ and propagation constants differing by dk (due to difference $d\lambda$ in their wavelengths) their equations can be written as,

$$y_1 = A \sin(\omega t - kx)$$

and $$y_2 = A \sin[(\omega + d\omega)t - (k + dk)x]$$

The resultant displacement 'y' at time 't' is,

$$y = y_1 + y_2$$

Using $$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right),$$

$$y = 2A\sin\left(\frac{2\omega + d\omega}{2}t - \frac{2k+dk}{2}x\right)\cos\left(\frac{d\omega}{2}\cdot t - \frac{dk}{2}\cdot x\right)$$

As $d\omega$ and dk are very small compared to ω and k respectively,

$$2\omega + d\omega \approx 2\omega$$

$$2k + dk \approx 2k$$

$$\therefore \quad y = 2A\sin(\omega t - kx)\cos\left(\frac{d\omega}{2}t - \frac{dk}{2}\cdot x\right)$$

The sine term in the above equation represents a wave of angular frequency ω and propagation constant k. The cosine term in the equation modulates this wave with angular frequency $\frac{d\omega}{2}$ and propagation constant $\frac{dk}{2}$ to produce wave groups (like phenomenon of 'beats' in sound waves) travelling with a velocity

$$v_g = \frac{d\omega}{dk}$$

which is the group velocity.

$$\therefore \quad v_g = \frac{d\omega/dv}{dk/dv} \quad \ldots (1)$$

The angular frequecny is given by

$$\omega = \frac{2\pi mc^2}{h}$$

As
$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\omega = \frac{2\pi m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0}{h} v\left(1-\frac{v^2}{c^2}\right)^{-3/2} \quad \ldots (2)$$

As
$$k = \frac{2\pi mv}{h}$$

$$\therefore \quad k = \frac{2\pi m_0 v}{h\sqrt{1-\frac{v^2}{c^2}}}$$

$$\frac{dk}{dv} = \frac{2\pi m_0}{h}\left(1-\frac{v^2}{c^2}\right)^{-3/2} \quad \ldots (3)$$

Substituting equations (2) and (3) in equation (1),

$$v_g = v \quad \ldots (4)$$

i.e., the De-Broglie wave group associated with moving particle travels with the same velocity as the particle.

Q.3 *Define phase velocity and group velocity. Derive an expression for De-Broglie wavelength from group velocity.* **VTU : Feb.-08, Marks 7, Aug.-11, Marks 6**

Ans. : **Phase velocity** (also known as wave velocity) is the velocity with which a particular phase of the wave propagates in a medium.

Mathematically, a wave group can be described interms of a superposition of individual waves of different wavelengths. The interference of these waves gives variation in amplitude defining the shape of the wave group. The velocity of this group of waves is known as **group velocity**.

The relation between group velocity v_g and phase velocity v_p is

$$v_g = v_p - \lambda \frac{dv_p}{d\lambda}$$

The group velocity v_g is same as the particle velocity v.

$$\therefore \quad v = v_p - \lambda \frac{dv_p}{d\lambda} \quad \ldots (1)$$

The above equation can be rewritten as,

$$v = \lambda^2 \left[\frac{v_p}{\lambda^2} - \frac{1}{\lambda} \frac{dv_p}{d\lambda} \right]$$

$$\therefore \quad v = -\lambda^2 \frac{d}{d\lambda} \left(\frac{v_p}{\lambda} \right)$$

As $v_p = \nu\lambda$ where ν = frequency,

$$\frac{v_p}{\lambda} = \nu$$

$$\therefore \quad v = -\lambda^2 \frac{d\nu}{d\lambda}$$

$$\frac{d\nu}{d\lambda} = \frac{-v}{\lambda^2} \quad \ldots (2)$$

The energy 'E' of the particle can be written as,

$$E = h\nu = \frac{1}{2} mv^2 + V$$

where V = Potential energy is constant

Differentiating with respect to λ,

$$h\frac{d\nu}{d\lambda} = mv\frac{dv}{d\lambda}$$

$$\therefore \quad \frac{dv}{d\lambda} = \frac{h}{mv}\frac{d\nu}{d\lambda}$$

Substituting from equation (2)

$$\frac{dv}{d\lambda} = \frac{h}{mv}\times\frac{-v}{\lambda^2}$$

$$\therefore \quad \frac{dv}{d\lambda} = \frac{-h}{m\lambda^2}$$

$$dv = \frac{-h}{m}\frac{d\lambda}{\lambda^2}$$

Integrating we get,

$$v = \frac{h}{m\lambda}$$

$$\therefore \quad \lambda = \frac{h}{mv}$$ which is the expression for De-Broglie wavelength.

Important Solved Examples

Example 1.1 *Calculate the kinetic energy of an electron whose De-Broglie wavelength is equal to that of a 10 keV photon. Take $h = 6.625 \times 10^{-34}$ J-s.* **VTU : Feb.-07, Marks 5**

Solution : The energy of a photon is given by,

$$E_p = \frac{hc}{\lambda_p}$$

$$\therefore \quad \lambda_p = \frac{hc}{E_p}$$

$$h = 6.625\times 10^{-34} \text{ J-s}$$

$$c = 3 \times 10^8 \text{ m/s}$$

$$E_p = 10 \text{ keV} = 10 \times 10^3 \times 1.6 \times 10^{-19} \text{ J}$$

$$\therefore \quad E_p = 1.6 \times 10^{-15} \text{ J}$$

$$\therefore \quad \lambda_p = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-15}}$$

$$\therefore \quad \lambda_p = 1.242 \times 10^{-10} \text{ m}$$

The wavelength 'λ_e' of an electron interms of its kinetic energy 'E' is given by,

$$\lambda_e = \frac{h}{\sqrt{2m_e E}}$$

As $\quad \lambda_e = \lambda_p = 1.242 \times 10^{-10}$ m

and $\quad m_e = 9.1 \times 10^{-31}$ kg,

$$E = \frac{h^2}{2m_e\lambda_e^2} = \frac{(6.625 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (1.242 \times 10^{-10})^2}$$

$$\therefore \quad \boxed{E = 1.563 \times 10^{-17} \text{ J} = 97.71 \text{ eV}}$$

Example 1.2 *A particle of mass 0.65 MeV/c^2 has a kinetic energy of 80 eV. Calculate the De-Broglie wavelength, group velocity and phase velocity of De-Broglie wave.*

VTU : Feb.-08, Jan.-13, Marks 5

Solution : By De-Broglie hypothesis,

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$h = 6.625 \times 10^{-34} \text{ J-s}$$

$$m = 0.65 \text{ MeV}/c^2$$

$$= 0.65 \times 1.6 \times 10^{-13} \text{ J}/c^2$$

$$= \frac{0.65 \times 1.6 \times 10^{-13}}{(3 \times 10^8)^2}$$

$$\therefore \quad m = 1.156 \times 10^{-30} \text{ kg}$$

$$E = 80 \text{ eV} = 80 \times 1.6 \times 10^{-19} \text{ J}$$

$$\therefore \quad E = 1.28 \times 10^{-17} \text{ J}$$

$$\therefore \quad \lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 1.156 \times 10^{-30} \times 1.28 \times 10^{-17}}}$$

$$\therefore \quad \lambda = 1.218 \times 10^{-10} \text{ m}$$

$$\therefore \quad \boxed{\lambda = 1.218 \text{ A}^\circ}$$

The group velocity 'v_g' is same as the particle velocity 'v',

By De-Broglie hypothesis,

$$\lambda = \frac{h}{mv}$$

$$\therefore \quad v_g = v = \frac{h}{m\lambda}$$

$$= \frac{6.625 \times 10^{-34}}{1.156 \times 10^{-30} \times 1.218 \times 10^{-10}}$$

$$\therefore \quad \boxed{v_g = 4.7 \times 10^6 \ m/s}$$

The phase velocity v_p is given by,

$$v_p = \frac{c^2}{v_g} = \frac{(3 \times 10^8)^2}{4.7 \times 10^6}$$

$$\therefore \quad \boxed{v_p = 1.91 \times 10^{10} \ m/s}$$

Important Multiple Choice Questions

Q.1 *The De-Broglie wavelength associated with an electron of mass 'm' and accelerated by a potential 'v' is _____.*

a) $\frac{h}{\sqrt{2mVe}}$ *b)* $\frac{\sqrt{2mVe}}{h}$ *c)* $\frac{h}{Vem}$ *d)* $\frac{h}{2Vem}$

VTU : Feb.-09, Aug.-10

(Ans. : a)

Q.2 *Electron behaves as waves as they can be _____.*

a) deflected by an electric field *b) diffracted by a crystal*

c) deflected by magnetic field *d) they ionize a gas*

VTU : Feb.-09, Aug.-10

(Ans. : b)

Q.3 *An electron and a proton are accelerated through same potential. The ratio of De-Broglie wavelength $\lambda_e/\lambda p$ is _____.*

a) 1 *b)* m_e/m_p *c)* m_p/m_e *d)* $\sqrt{\frac{m_p}{m_e}}$

VTU : Aug.-09

(Ans. : d)

Q.4 *In black body radiation spectrum, the maximum energy peaks shift towards the shorter wavelength side with increase in temperature. This confirms _____.*

a) Stefan's law *b) Wien's law*

c) Rayleigh-Jean's law *d) Planck's law*

VTU : Aug.-09

(Ans. : b)

Q.5 *The group velocity of the particle is 3×10^6 m/s whose phase velocity is _____.*

a) 6.06×10^6 m/s *b) 3×10^{10} m/s* *c) 3×10^6 m/s* *d) 1.5×10^{10} m/s*

VTU : Aug.-09

(Ans. : b)

Q.6 *Wien's law is deduced from Planck's radiation formula under the condition of _____.*

a) very small wavelength and temperature

b) large wavelength and temperature

c) small wavelength and high temperature

d) large wavelength and small temperature

VTU : Feb.-10

(Ans. : a)

Q.7 *The compton wavelength is given by _____.*

a) $\frac{h}{m_0c^2}$ *b)* $\frac{h^2}{m_0c^2}$ *c)* $\frac{h}{m_0c}$ *d)* $\frac{h^2}{2m_0c}$

VTU : Feb.-10, June-12

(Ans. : c)

Q.8 *Which of the following can be used to determine De-Broglie wavelength associated with a particle ?*

a) $\frac{h}{\sqrt{2mE}}$ *b)* $\frac{h}{mv}$ *c)* $\frac{h}{\sqrt{2meV}}$ *d) All of these*

VTU : Feb.-10

(Ans. : d)

Q.9 *If the group velocity of a particle is 3×10^6 m/s, its phase velocity is _____.*

a) 100 m/s *b) 3×10^6 m/s* *c) 3×10^8 m/s* *d) 3×10^{10} m/s*

VTU : Feb.-10, Jan.-12

(Ans. : d)

Q.10 *If electron, proton, neutron and α-particle have the same De-Broglie wavelength, the particle with largest velocity will be _____.*

a) electron *b) proton* *c) neutron* *d) α-particle*

(Ans. : a)

Q.11 *If an electron, proton, neutron and α-particle have the same velocity, the particle which has the longest wavelength is _____.*

a) electron *b) proton* *c) neutron* *d) α- particle*

(Ans. : a)

Q.12 *The De-Broglie wavelength of a particle does not depend upon its _____.*

a) mass *b) velocity* *c) acceleration* *d) kinetic energy*

(Ans. : c)

Q.13 *A proton and an α-particle have the same kinetic energy. If the mass of the α-particle is 4 times the mass of proton, the ratio of De-Broglie wavelength of α-particle to that of proton is _____.*

a) 1 : 2 *b) 2 : 1* *c) 1 : 4* *d) 4 : 1*

(Ans. : a)

Q.14 *Photoelectric effect establishes _____.*

a) wave nature of light *b) particle nature of light*

c) dual nature of light *d) none of the above.* **VTU : June-12**

(Ans. : b)

Q.15 *The law which describes the black body radiation completely is _____.*

a) Planck's law *b) Stefan's law* *c) Wien's law* *d) Rayleigh-Jeans law*

VTU : Jan.-12

(Ans. : a)

Q.16 *Rayleigh-Jeans law is derived from _____.*

a) electromagnetic wave theory of light *b) quantum theory of light*

c) dual nuture of light *d) none of the above*

(Ans. : a)

Q.17 *If the group velocity of De-Broglie waves associated with a particle is 3×10^4 m/s, the velocity of the particle is _____.*

a) 3×10^8 m/s *b) 3×10^{12} m/s* *c) 3×10^4 m/s* *d) 30 m/s*

VTU : June-12

(Ans. : c)

Q.18 *The De-Broglie wavelength of an electron at rest will be _____.*

a) 0 *b) ∞* *c) 100 m* *d) 1 km*

(Ans. : b)

Q.19 *The De-Broglie wavelength of an electron accelerated through a potential difference of 100 V is _____.*

a) 1 A° *b) 10 A°* *c) 1.227 A°* *d) 12.27 A°*

(Ans. : c)

Q.20 *In compton effect, the wavelength same as incident wavelength is observed along with longer wavelength at all scattering angles due to _____.*

a) elastic collision of x-ray photons with free electrons

b) elastic collision of x-ray photons with bound electrons

c) inelastic collision of x-ray photons with free electrons

d) inelastic collision of x-ray photons with bound electrons

(Ans. : b)

Q.21 *In photoelectric effect, the maximum kinetic energy of photoelectron depends on _____.*

a) wavelength of incident light *b) velocity of incident light*

c) intensity of incident light

d) time for which light is incident on the surface

(Ans. : a)

Q.22 *A perfectly black body at low temperature _____.*

a) absorbs all the incident radiation

b) absorbs only certain wavelengths from the incident radiation

c) reflects all the incident radiation *d) absorbs more radiation than it emits*

(Ans. : d)

Q.23 *The compton wavelength depends only on _____.*

a) wavelength of incident x-rays *b) frequency of incident x-rays*

c) velocity of incident x-ray *d) mass of the scatterer*

(Ans. : d)

Q.24 *Green light incident on a surface releases photoelectrons from the surface. If now blue light is incident on the same surface, the velocity of electrons _____.*

a) increases *b) decreases* *c) remains same* *d) becomes zero*

VTU : Feb.-11

(Ans. : a)

Q.25 *Rayleigh-Jean's theory of radiations agree with experimental results for _____.*

a) all wavelengths *b) shorter wavelengths only*

c) longer wavelengths only *d) middle order wavelengths only*

VTU : Feb.-11

(Ans. : c)

Q.26 *The De-Broglie wavelength of an electron accelerated to a potential difference of 100 volts is _____.*

a) 1.2 A° *b) 10 A°* *c) 100 A°* *d) 12 A°*

VTU : Feb.-11

(Ans. : a)

Q.27 *The wave nature associated with electrons in motion was verified by _____.*

a) photoelectric effect *b) Compton effect*

c) diffraction by crystals *d) Ramon effect*

VTU : Feb.-11

(Ans. : c)

Q.28 *In Compton Effect, the wavelength of the x-rays scattered at an angle $\theta > 0$ _____.*

a) increases *b) doesn't change* *c) decreases* *d) none of these*

VTU : Aug.-11, June-13

(Ans. : a)

Q.29 *K_e, K_P and K_α an respective kinetic energy of an $\bar{e}$, a proton and α - particle of same De-Broglie wavelength, then _____.* **VTU : Aug.-11**

a) $K_e > K_P > K_\alpha$ *b)* $K_e > K_P < K_\alpha$ *c)* $K_e < K_P < K_\alpha$ *d)* $K_e = K_P = K_\alpha$

(Ans. : a)

Q.30 *The heavier of the particles has smallest De-Broglie wavelength when both of them _____.* **VTU : Aug.-11**

a) move with same speed *b) move with same momentum*

c) move with same kinetic energy *d) none of these*

(Ans. : a)

Q.31 *Matter waves are not electromagnetic waves because _____.* **VTU : Aug.-11**

a) they move with variable velocity *b) depend on charge*

c) move with constant velocity *d) none of these*

(Ans. : a)

Q.32 *The wavelength (λ) associated with a particle of mass, m, moving with velocity V is given by _____.*

a) $\lambda = \frac{h}{mV}$ *b)* $\lambda = \frac{mV}{h}$ *c)* $\lambda = \frac{hV}{m}$ *d)* $\lambda = \frac{m}{hV}$

VTU : Jan.-12

(Ans. : a)

Q.33 *Ultraviolet catastrophe is the failure of Rayleigh-Jeans law in explaining the black-body radiation for wavelength _____.*

a) equal to that in visible range *b) longer than that of violet light*

c) shorter than that of violet light *d) None of these* **VTU : June-12**

(Ans. : c)

Q.34 *The law which failed to account for shorter wavelength region of black body radiation spectrum is, _____.*

a) Wein's law *b) Rayleigh-Jean's law*

c) Planck's law *d) Maxwell's law* **VTU : Jan.-13**

(Ans. : b)

Q.35 *The de-Broglie wavelength of a particle at rest is _____.*

a) zero *b) infinite* *c) h/p* *d) h/v*

VTU : Jan.-13

(Ans. : b)

Q.36 *If group velocity of particle is 4.7×10^6 m/s then its phase velocity is _____.*

a) 6×10^9 *m/s* *b)* 4.7×10^6 *m/s* *c)* 9.4×10^6 *m/s* *d)* 1.91×10^{10} *m/s*

VTU : Jan.-13

(Ans. : d)

Q.37 *The particle velocity of wave is equal to _____.* **VTU : Jan.-13**

a) group velocity *b) phase velocity* *c) velocity of light* *d) velocity of sound*

(Ans. : a)

Q.38 *If red and blue stars emits radiations of continuous wavelengths, then according to Wien's displacement law _____.*

a) blue star is hotter than red star *b) red star is hotter than blue star*

c) both stars are at same temperature *d) difficult to conclude* **VTU : June-13**

(Ans. : a)

Q.39 *The expression for de-Broglie wavelength for an electron under an accelerating potential V is _____.*

a) $\frac{12.26}{\sqrt{V}}$ *m* *b)* $\frac{12.26}{\sqrt{V}}$ $\mathring{A}$ *c)* $\frac{12.26}{\sqrt{V}}$ *nm* *d)* $\frac{12.26}{\sqrt{V}}$ μm

VTU : June-13

(Ans. : b)

Q.40 *A particle moves with velocity* 3×10^6 ms^{-1}. *The wavelength associated with it is 1 nm. Then group velocity of the particle is _____.*

a) $3 \times 10^8 ms^{-1}$ *b)* 3×10^{10} ms^{-1} *c)* 3×10^6 ms^{-1} *d)* 1.5×10^6 ms^{-1}

VTU : June-13

(Ans. : c)

Q.41 *Compton effect establishes _____ nature of electromagnetic radiation.*

a) wave *b) particle* *c) dual* *d) none of the above*

(Ans. : b)

Q.42 *If only the frequency of light incident on a photoelectric material is increased beyond the threshold frequency, _____ increases.*

a) photoelectric current *b) work function of the material*

c) maximum kinetic energy of emitted electrons

d) none of the above

(Ans. : c)

Q.43 *If only the intensity of light incident on a photoelectric material is increased beyond the threshold frequency, _____ increases.*

a) photoelectric current *b) work function of the material*

c) maximum kinetic energy of emitted electrons

d) none of the above

(Ans. : a)

❑❑❑

2 Quantum Mechanics

Chapter at a Glance

- The Heisenberg's uncertainty principle states that it is impossible to determine both the exact position and the exact momentum of a particle at the same time. The product of uncertainties in these quantities is always greater than or equal to $\frac{h}{4\pi}$.

$$\Delta x \, \Delta p_x \geq \frac{h}{4\pi}$$

- Uncertainty principle applied to energy and time :

$$\Delta E \, \Delta t \geq \frac{h}{4\pi}$$

- According to Max Born's interpretation, the probability of finding the particle described by the wave function ψ at a point (x, y, z) in space at time 't' proportional to the value of $|\psi|^2$ at that point at time 't'.

- Time Independent Schroedinger Wave Equation in One Dimension :

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E - V)\,\psi = 0$$

- The energy eigen values of a particle in a rigid box are given by

$$E_n = \frac{n^2 h^2}{8mL^2}$$

where, $n = 1, 2, 3, \ldots$

The eigen functions are given by

$$\psi_n = \sqrt{\frac{2}{L}}\, sin\left(\frac{n\pi x}{L}\right)$$

Important Theory Questions and Answers

Q.1 *Explain Heisenberg's uncertainty principle. Based on this, show the non-existence of electrons inside the nucleus.* **VTU : Feb.-08, Marks 7, June-10, 12, Marks 8**

Ans. : Describing a moving particle as a wave group introduces uncertainty in the measurement of particle properties like position and momentum. This uncertainty is not due to any inadequacy in the measuring instruments but is inherent in wave nature. The position of particle becomes uncertain when it is described as a wave group. The particle may be located anywhere within the wave group but its exact location within the wave group is not known. Narrower the wave group, smaller is the uncertainty in its position as illustrated in Fig. 2.1 (a) and (b).

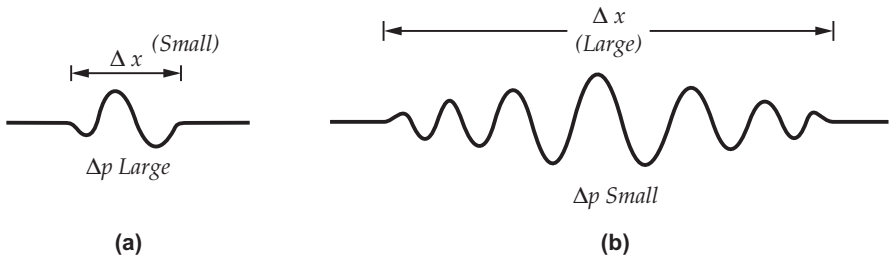


Fig. 2.1

For a purely sinusoidal wave $\psi(x)$, the Fourier transform $g(\omega)$ gives amplitude only at a single frequency as shown in Fig. 2.2 (a). For a wave group a broader range of frequencies are present as shown in Fig. 2.2 (b) and for a narrow pulse shown in Fig. 2.2 (c), a very large number of frequencies are present. Hence, if the wave group is narrower, it will consist of larger number of frequencies and hence wavelengths due to which the wavelength becomes more uncertain. As $\lambda = \frac{h}{p}$, an uncertainty in wavelength leads to uncertainty in momentum. Thus for broader wave groups, there is smaller uncertainty in wavelength and momentum, whereas for narrower wave groups there is larger uncertainty in wavelength and momentum.

From the above discussion, we can conclude that for a narrow wave group there is small uncertainty in position but large uncertainty in momentum, whereas for a broader wave group there is larger uncertainty in position and smaller uncertainty in momentum.

The Heisenberg's uncertainty principle states that it is impossible to determine both the exact position and the exact momentum of a particle at the same time. The product of uncertainties in these quantities is always greater than or equal to $\frac{h}{4\pi}$.

$$\Delta x \, \Delta p_x \geq \frac{h}{4\pi}$$

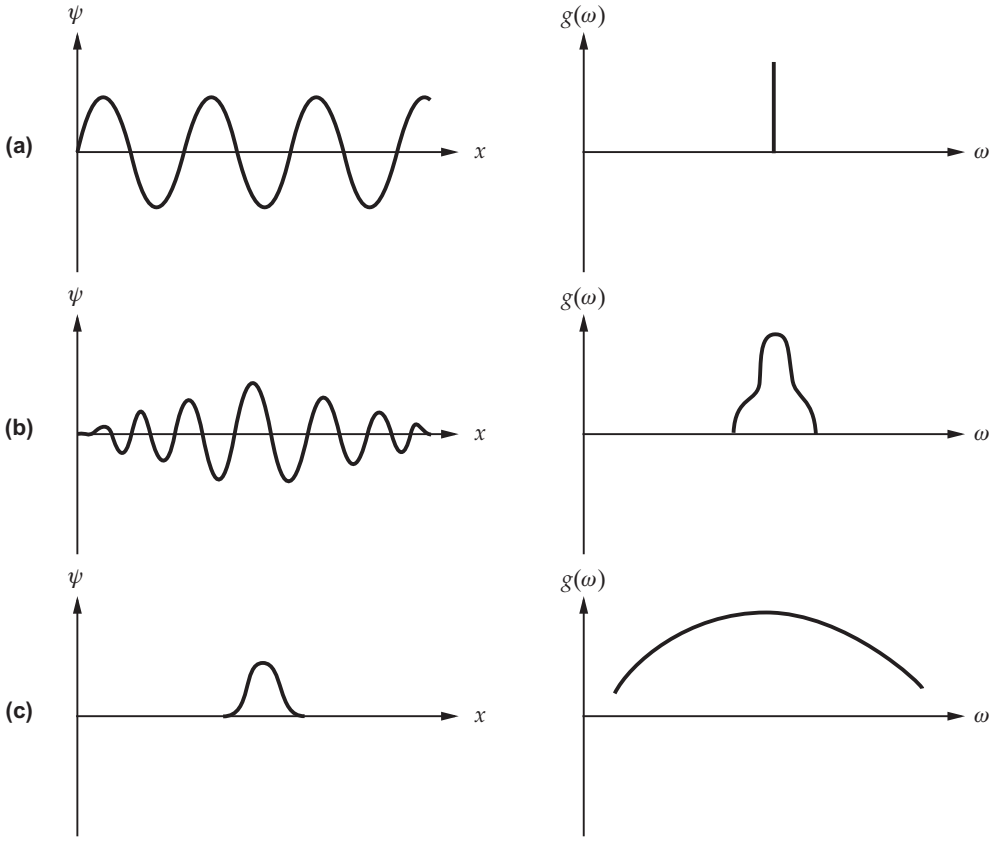


Fig. 2.2

Non-existence of electron in the nucleus

If an electron is confined to the nucleus which has a radius of the order of 10^{-14} m, the maximum uncertainty in position of the electron will be of the order of the radius.

$$\therefore \quad (\Delta x)_{max} = 10^{-14}\ m$$

By Heisenberg uncertainty principle,

$$(\Delta x)_{max}\,(\Delta p)_{min} = \frac{h}{4\pi}$$

$$\therefore \quad (\Delta p)_{min} = \frac{h}{4\pi(\Delta x)_{max}}$$

$$= \frac{6.63 \times 10^{-34}}{4\pi \times 10^{-14}}$$

$$\therefore \quad (\Delta p)_{min} = 5.276 \times 10^{-21}\ kg \cdot m/s$$

The momentum of electron has to be atleast comparable in magnitude to this uncertainty.

$$\therefore p_{min} \simeq (\Delta p)_{min} = 5.276 \times 10^{-21} \ kg \cdot m/s$$

The equation for energy from theory of relativity is

$$E = \sqrt{p^2c^2 + m_0^2 c^4}$$

Here, $m_0^2 c^4 << p^2c^2$

$$\therefore \quad E = pc$$

$$\therefore \quad E_{min} = 5.276 \times 10^{-21} \times 3 \times 10^8$$

$$= 1.583 \times 10^{-12} \ J$$

$$\therefore \quad E_{min} = 9.9 \text{ MeV}$$

Thus, for an electron to exist in the nucleus, its energy must be atleast 9.9 MeV. Experimentally, it has been observed during beta decay that the β-particles (electrons emitted by the nucleus) never have energy exceeding about 4 MeV. As the minimum energy of an electron in a nucleus is 9.9 MeV, the electron can never exist in a nucleus. In fact the β-particles are electrons which are produced in the nucleus due to decay of neutrons.

Q.2 *Set up time independent Schroedinger wave equation.*

VTU : Feb.-09, 12, 13, Aug.-09, Marks 6

Ans. : The general differential equation of a wave travelling in x-direction with velocity 'u' having wave function ψ is given by,

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{u^2} \frac{\partial^2 \psi}{\partial t^2} \quad \text{... (1)}$$

The general solution of equation (1) is of the form

$$\psi = \psi_0 \, e^{i(kx - \omega t)} \quad \text{... (2)}$$

where ψ_0 is a constant.

Differentiating equation (2) partially with respect to 't' twice we get,

$$\frac{\partial \psi}{\partial t} = (-i\omega) \psi_0 \, e^{i(kx - \omega t)}$$

$$\frac{\partial^2 \psi}{\partial t^2} = (-i\omega)^2 \psi_0 \, e^{i(kx - \omega t)} = -\omega^2 \psi_0 \, e^{i(kx - \omega t)}$$

$$\therefore \quad \frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi$$

Substituting in equation (1),

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{\omega^2}{u^2} \psi \quad \text{... (3)}$$

$$\omega = 2\pi \nu = \frac{2\pi u}{\lambda}$$

$$\therefore \quad \frac{\omega^2}{u^2} = \frac{4\pi^2}{\lambda^2}$$

By De Broglie hypothesis, $\lambda = \frac{h}{p}$

$$\therefore \quad \frac{\omega^2}{u^2} = \frac{4\pi^2 p^2}{h^2} \quad \text{... (4)}$$

The total energy 'E' is a sum of the kinetic energy $\frac{1}{2} mv^2$ and potential energy V.

$$E = \frac{1}{2} mv^2 + V$$

$$= \frac{1}{2} \frac{m^2 v^2}{m} + V$$

$$\therefore \quad E = \frac{p^2}{2m} + V \qquad as \;\; p = mv$$

$$\therefore \quad p^2 = 2m\,(E - V)$$

Substituting in equation (4),

$$\frac{\omega^2}{u^2} = \frac{8\pi^2 m}{h^2} (E - V)$$

Substituting in equation (3),

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{8\pi^2 m}{h^2} (E - V) \psi$$

$$\therefore \quad \frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0$$

The above equation is used when the potential energy is constant in time but varies in space. Hence the wave function will also depend only on space co-ordinates. The partial derivative of ψ with respect to x can therefore be replaced by the total derivative.

$$\therefore \frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-V)\,\psi = 0 \qquad \text{... (5)}$$

Equation (5) which is used for potential energy distributions that remain constant with time and only vary with x and hence gives solution for ψ as function of x only is known as the Schroedinger time independent wave equation.

A quantity $\frac{h}{2\pi}$ occurs very frequently in more advanced applications. Hence $\frac{h}{2\pi}$ is replaced by $\hbar$ in equation (5).

$$\therefore \frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\,\psi = 0$$

Q.3 *Find the eigen functions and eigen values for a particle in one dimensional potential well of infinite height and discuss the solutions.*

VTU : Aug.-07, Marks 11, Feb.-11, Marks 8

Ans. : The physical problem of particle confined between two rigid walls can be converted into a problem of potential distribution by specifying the potential energy of the particle to be infinite at and beyond the walls. i.e.,

$$V = \infty \text{ for } x \le 0 \text{ and for } x \ge L$$

The potential energy of the particle is constant within the box which can be taken to be zero for convenience i.e.,

$$V = 0 \text{ for } 0 < x < L$$

The potential energy distribution is shown in Fig. 2.3 (b). It is as if the particle is inside an infinite potential well. As the particle does not exist at the walls and beyond them,

$$\psi = 0 \text{ for } x \le 0 \text{ and } x \ge L$$

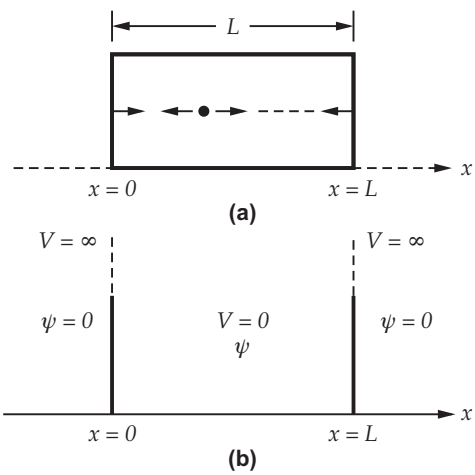


Fig. 2.3

The wave function ψ exists only for $0 < x < L$. Schroedinger's time independent wave equation is

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-v)\,\psi = 0 \qquad \text{... (1)}$$

Substituting $V = 0$ for $0 < x < L$ in equation (1),

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2} E\psi = 0$$

Let $$K^2 = \frac{8\pi^2 mE}{h^2} \quad \ldots (2)$$

$$\therefore \quad \frac{d^2\psi}{dx^2} + K^2\psi = 0 \quad \ldots (3)$$

The general solution of equation (3) is

$$\psi = A \sin Kx + B \cos Kx \quad \ldots (4)$$

where A and B are arbitrary constants which are to be determined using boundary conditions.

The first boundary condition is

$$\psi = 0 \text{ at } x = 0$$

$\therefore$ From equation (4),

$$0 = A \sin 0 + B \cos 0$$

$$B = 0$$

$\therefore$ From equation (4)

$$\psi = A \sin Kx \quad \ldots (5)$$

The second boundary condition is

$$\psi = 0 \text{ at } x = L$$

$\therefore$ From equation (5),

$$0 = A \sin KL$$

$A \neq 0$ as for $A = 0$, $\psi = 0$ for all values of x which will mean that the particle does not exist inside the box.

$$\therefore \quad \sin KL = 0$$

$$\therefore \quad KL = n\pi ; \; n = 1, 2, 3, \ldots$$

$n \neq 0$ as $n = 0 \Rightarrow \psi = 0$ for all values of x which is not possible.

$$\therefore \quad K = \frac{n\pi}{L} \quad \ldots (6)$$

From equations (2) and (5),

$$\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{L^2}$$

As energy 'E' depends on 'n', we use suffix 'n' for 'E'.

$$\therefore \qquad E_n = \frac{n^2 h^2}{8mL^2} \qquad \ldots (7)$$

where, $n = 1, 2, 3, \ldots$

From the above equation, the smallest value of energy that the particle can have is

$$E_1 = \frac{h^2}{8mL^2}$$

which is non-zero. This contradicts classical mechanics, according to which the particle can have zero energy.

The other possible values of energy are $E_2 = \frac{4h^2}{8mL^2}$, $E_3 = \frac{9h^2}{8mL^2}$, etc.

These energy values are discrete. They are not continuous as expected from classical mechanics. Thus, according to quantum mechanics, the particle inside a rigid box cannot have all values of energy, but only those discrete values which are given by equation (7). These discrete energy values are known as energy eigen values. These energy levels are shown in Fig. 2.4.

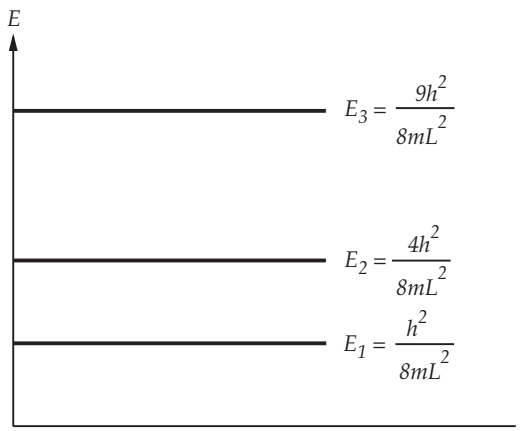


Fig. 2.4

In a similar situation in three dimensions, an electron in an atom has descrete energy levels. During transition from higher level to lower level, the energy difference is given out and for transition from lower level to higher, energy equivalent to the energy difference is absorbed.

Wave function : Substituting equation (6) in equation (5), we get,

$$\psi = A \sin\left(\frac{n\pi x}{L}\right) \qquad \ldots (8)$$

The complex conjugate of ψ is

$$\psi^* = A \sin\left(\frac{n\pi x}{L}\right)$$

To normalize the wave function, we find $\int_0^L \psi\psi^* dx$

$$\int_0^L \psi\psi^* dx = \int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx$$

$$= A^2 \int_0^L \left[\frac{1-\cos\left(\frac{n\pi x}{L}\right)}{2}\right] dx$$

$$= \frac{A^2}{2}\left[x - \frac{\sin\left(\frac{n\pi x}{L}\right)}{\left(\frac{n\pi}{L}\right)}\right]_0^L$$

$$= \frac{A^2}{2}[L-0]$$

$$\therefore \quad \int_0^L \psi\psi^* dx = \frac{A^2 L}{2}$$

Let the R.H.S. be N^2 i.e.,

$$N^2 = \frac{A^2 L}{2}$$

or $$N = A\sqrt{\frac{L}{2}}$$

The normalized wave function ψ_n is obtained using

$$\psi_n = \frac{\psi}{N}$$

$$\therefore \quad \psi_n = \frac{A \sin\left(\frac{n\pi x}{L}\right)}{\left(A\sqrt{\frac{L}{2}}\right)}$$

$$\therefore \quad \psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \quad \ldots (9)$$

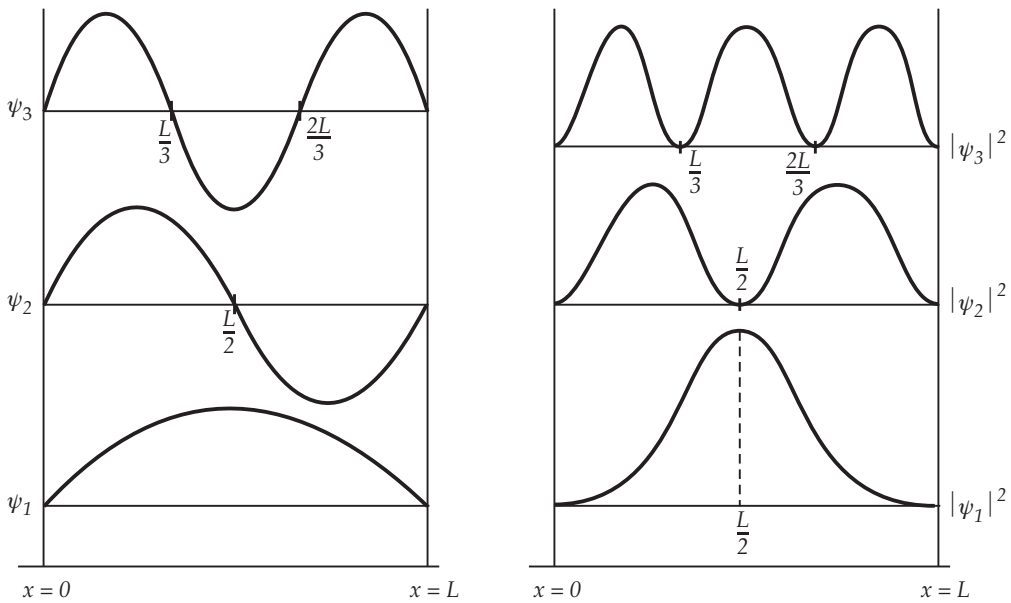
These normalized wave functions are called eigen functions.

$$\psi_n^* = \sqrt{\frac{2}{L}}\, sin\left(\frac{n\pi x}{L}\right)$$

The probability function is

$$P(x) = |\psi_n|^2 = \psi_n \psi_n^* = \frac{2}{L} sin^2\left(\frac{n\pi x}{L}\right) \quad \ldots (10)$$

The wave function and probability functions for $n = 1, 2, 3$ are shown in Fig. 2.5.



(a) Wave function **(b) Probability function**

Fig. 2.5

A particle having the lowest energy E_1 has wave function ψ_1 for which the probability of finding the particle is maximum at the centre of the box. For a particle of energy E_2, the probability of finding the particle is zero at the centre. Thus quantum mechanics predicts certain forbidden regions for the particle with a certain energy. Such situations are encountered with electrons in atoms. This is not predicted by classical mechanics in which there is same probability of finding the particle throughout the region.

Important Solved Examples

Example 2.1 *A spectral line of wavelength 4000 A° has a width of 8×10^{-5} A°. Evaluate the minimum time spent by the electrons in the upper energy state between the excitation and de-excitation processes.* **VTU : Aug.-07, Jan.-13, Marks 5**

Solution : By Heisenberg's uncertainty principle,

$$\Delta E \, \Delta t \geq \frac{h}{4\pi}$$

As
$$E = \frac{hc}{\lambda},$$

$$|\Delta E| = \frac{hc}{\lambda^2} \Delta\lambda$$

$$\therefore \quad \frac{hc}{\lambda^2} \Delta\lambda \, \Delta t \geq \frac{h}{4\pi}$$

$$\therefore \quad \Delta t \geq \frac{\lambda^2}{4\pi c \Delta\lambda}$$

$$\lambda = 4000 \text{ A}^\circ = 4 \times 10^{-7} \text{ m}$$

$$c = 3 \times 10^8 \text{ m/s}$$

$$\Delta\lambda = 8 \times 10^{-5} \text{ A}^\circ = 8 \times 10^{-15} \text{ m}$$

$$\frac{\lambda^2}{4\pi c \Delta\lambda} = \frac{(4 \times 10^{-7})^2}{4\pi \times 3 \times 10^8 \times 8 \times 10^{-15}}$$

$$= 5.305 \times 10^{-9}$$

$$\therefore \quad \boxed{\Delta t \geq 5.305 \times 10^{-9} \text{ s}}$$

Example 2.2 *Estimate the time spent by an atom in the excited state during the excitation and de-excitation processes, when a spectral line of wavelength 546 nm and width 10^{-14} m is emitted.* **VTU : Feb.-11, June-13, Marks 4**

Solution : By Heisenberg's uncertainty principle,

$$\Delta E \, \Delta T \geq \frac{h}{4\pi}$$

At
$$E = \frac{hc}{\lambda}$$

$$|\Delta E| = \frac{hc}{\lambda^2}\Delta\lambda$$

$$\therefore \quad \frac{hc}{\lambda^2}\Delta\lambda\ \Delta t \geq \frac{h}{4\pi}$$

$$\therefore \quad \Delta t \geq \frac{\lambda^2}{4\pi c\,\Delta\lambda}$$

$$\lambda = 546 \text{ nm} = 546\times10^{-9}\ m$$

$$c = 3\times10^{8}\ m/s$$

$$\Delta\lambda = 10^{-14}\ m$$

$$\therefore \quad \Delta t \geq \frac{(546\times10^{-9})^2}{4\pi\times3\times10^{8}\times10^{-14}}$$

$$\therefore \quad \boxed{\Delta t \geq 7.91\times10^{-9}\ s}$$

Example 2.3 *A quantum particle confined to one dimensional box of width 'a' is in its first excited state. What is the probability of finding the particle over an interval of* $\left(\frac{a}{2}\right)$ *marked symmetrically at the centre of the box ?* **VTU : Feb.-09, Aug.-09, 11, Marks 6**

Solution : For a box of width '*a*', the normalized eigen functions are given by,

$$\psi_n = \sqrt{\frac{2}{a}}\ sin\left(\frac{n\pi x}{a}\right)$$

The complex conjugate is

$$\psi_n^* = \sqrt{\frac{2}{a}}\ sin\left(\frac{n\pi x}{a}\right)$$

For the first excited state, n = 2

$$\psi_1 = \sqrt{\frac{2}{a}}\ sin\left(\frac{2\pi x}{a}\right)$$

$$\psi_1^* = \sqrt{\frac{2}{a}}\ sin\left(\frac{2\pi x}{a}\right)$$

For an interval $\left(\frac{a}{2}\right)$ symmetric about the centre, the probability of finding the particle is

$$P = \int_{a/4}^{3a/4} \psi_1 \psi_1^* \, dx$$

$$= \int_{a/4}^{3a/4} \frac{2}{a} \sin^2\left(\frac{2\pi x}{a}\right) dx$$

$$= \frac{2}{a} \int_{a/4}^{3a/4} \left[\frac{1-\cos\left(\frac{4\pi x}{a}\right)}{2}\right] dx$$

$$= \frac{1}{a}\left[x - \frac{\sin\left(\frac{4\pi x}{a}\right)}{\left(\frac{4\pi}{a}\right)}\right]_{a/4}^{3a/4}$$

$$= \frac{1}{a}\left\{\left[\frac{3a}{4} - \frac{a}{4\pi}\sin(3\pi)\right] - \left[\frac{a}{4} - \frac{a}{4\pi}\sin(\pi)\right]\right\}$$

$$= \frac{1}{a}\left\{\frac{3a}{4} - \frac{a}{4}\right\} = \frac{1}{a} \times \frac{2a}{4}$$

$$\therefore \quad \boxed{P = 0.5}$$

Important Multiple Choice Questions

Q.1 *The product of uncertainty between angular momentum and angular displacement is*

a) $\geq \frac{h}{2\pi}$ b) $\geq \frac{h}{4\pi}$ c) $\frac{h}{2\pi}$ d) $\leq \frac{h}{4\pi}$

VTU : Feb.-09

(Ans. : b)

Q.2 *Kinetic energy of electron accelerated by a voltage of 50 volts is _____ .*

a) 50 eV b) 10 eV c) 5 eV d) 15 eV

VTU : Feb.-09

(Ans. : a)

Q.3 *The energy of the lowest state in one dimensional potential box of length 'a' is _____.*

a) zero b) $\frac{2h^2}{8ma^2}$ c) $\frac{h^2}{8ma^2}$ d) $\frac{h}{8ma^2}$

VTU : Feb.-09

(Ans. : c)

Q.4 *The wave function for the motion of particles in one dimensional potential box of length 'a' is given by* $\psi_n = D \sin\frac{n\pi x}{a}$, *where D is the normalization constant. The value of D is* _____ .

a) $\frac{1}{a}$ b) $\sqrt{\frac{2}{a}}$ c) a d) $\sqrt{\frac{a}{2}}$

VTU : Feb.-09, June-10

(Ans. : b)

Q.5 *Wave function associated with a material particle is* _____ .

a) Single valued b) Finite

c) Continuous d) All the above

VTU : Aug.-09, June-10

(Ans. : d)

Q.6 *According to Max Born approximation* $|\psi|^2$ *represents* _____ .

a) Energy density b) Particle density

c) Probability density d) Charge density

VTU : Aug.-09, June-10

(Ans. : c)

Q.7 *An electron has a speed of 100 m/s accurate to 0.05 %. The uncertainty in its position is* _____ .

a) 0.01 m b) 0.0115 m c) 0.024 m d) 0.04 m

VTU : Aug.-09, 11

(Ans. : b)

Q.8 *An electron is moving in a box of length 'a'. If* ψ_1 *is the wave function at* $x = \frac{a}{4}$ *with n = 1 and* ψ_2 *at x = a for n = 2, then* $\frac{\psi_2}{\psi_1}$ *is* _____ .

VTU : Aug.-09, 11

a) $\frac{\sqrt{2}}{a}$ b) $\frac{\sqrt{a}}{2}$ c) 0 d) ∞

(Ans. : c)

Q.9 *The lowest quantized energy of a particle of mass 'm' in a box of length L is given by* _____ .

a) zero b) $\frac{h^2}{8mL^2}$ c) $\frac{2h^2}{8mL^2}$ d) $\frac{h^2}{2mL^2}$

VTU : Aug.-09

(Ans. : b)

Q.10 *The normalization of wave function is always possible if* _____ .

a) $\int_{-\infty}^{\infty} \psi\psi^* \, dx$ = Infinite b) $\int_{-\infty}^{\infty} \psi\psi^* \, dx$ = Finite

c) $\int_{-\infty}^{\infty} \psi\psi^* \, dx = 0$ d) All of these

VTU : Feb.-10

(Ans. : b)

Q.11 *Schroedinger's time independent wave equation is applicable for the particles with _____.*

a) Constant energy b) Variable energy

c) Only constant potential energy d) All of these

VTU : Feb.-10

(Ans. : c)

Q.12 *The ground state energy of an electron in an infinite well is 5.6 MeV. If the width of the well is doubled, the ground state energy is _____ .*

a) 9.92×10^{-23} J b) 4.48×10^{-22} J

c) 2.24×10^{-22} J d) None of these

VTU : Feb.-10

(Ans. : d)

Q.13 *The wave function is acceptable wave function if it is _____ .*

a) Finite everywhere b) Continuous everywhere

c) Single valued everywhere d) All of these

(Ans. : d)

Q.14 *The energy required to be supplied to a particle in a one dimensional infinite potential well to raise it from its ground state to the second excited state is _____ .*

a) $\frac{4h^2}{8mL^2}$ b) $\frac{9h^2}{8mL^2}$ c) $\frac{h^2}{mL^2}$ d) $\frac{2h^2}{8mL^2}$

(Ans. : c)

Q.15 *The energy given out by a particle in a one dimensional infinite potential well when it falls from the second excited state to its ground state is _____ .*

a) $\frac{4h^2}{8mL^2}$ b) $\frac{9h^2}{8mL^2}$ c) $\frac{h^2}{mL^2}$ d) $\frac{2h^2}{8mL^2}$

(Ans. : c)

Q.16 *The energy difference between the nth and the $(n+1)^{th}$ energy levels for a particle in a rigid box of length 'L' is _____ .*

a) $\frac{h^2}{8mL^2}$ b) $(2n+1)\frac{h^2}{8mL^2}$ c) $(2n-1)\frac{h^2}{8mL^2}$ d) $(n+1)\frac{h^2}{8mL^2}$

(Ans. : b)

Q.17 *If an electron is confined to a box of length 'L' and 'L' is decreased, the uncertainty in velocity of electron _____ .*

a) Increases b) Decreases

c) Remains constant d) First increases and then decreases

(Ans. : a)

Q.18 *For a particle in an infinite potential well in its first excited state, the probability of finding the particle at the centre of the box is _____ .* **VTU : Aug.-11**

a) 0 *b) 0.25* *c) 0.5* *d) 0.1*

(Ans. : a)

Q.19 *For a particle in an infinite potential well in its first excited state, the probability of finding the particle is maximum at _____ .*

a) $x = 0$ *b)* $x = L$ *c)* $x = \frac{L}{2}$ *d)* $x = \frac{L}{4}$ *and* $\frac{3L}{4}$

(Ans. : d)

Q.20 *The uncertainties in position and momentum of a particle in any simultaneous measurement of these quantities are due to _____ .*

a) Limitations of measuring instruments *b) Human limitations*

c) both (a) and (b) *d) None of the above*

(Ans. : d)

Q.21 *If the wave group is narrow, then there is _____ .* **VTU : June-12**

a) Large uncertainty in momentum *b) Small uncertainty in momentum*

c) No uncertainty in momentum *d) None of the above*

(Ans. : a)

Q.22 *If uncertainty in momentum is large, the uncertainty in wavelength is _____ .*

VTU : June-12

a) Small *b) Large* *c) Zero* *d) None of the above*

(Ans. : b)

Q.23 *For a particle in a one dimensional rigid box, the energy difference between adjacent levels _____ .*

a) Increases with n *b) Decreases with n*

c) Remains constant *d) None of the above*

(Ans. : a)

Q.24 *Which of the following statements is correct regarding the energy eigen values of a particle in a rigid box ?*

a) The minimum energy is not zero

b) The energy eigen values are discrete

c) Both (a) and (b)

d) Neither (a) nor (b)

(Ans. : c)

Q.25 *An electron, a proton and an alpha particle are enclosed in three rigid boxes of the same width. The energy levels will be closer together for _____ .* **VTU : June-12**

a) Electron *b) Proton* *c) Alpha particle* *d) None of the above*

(Ans. : c)

Q.26 *Which of the following functions cannot be accepted as solutions for Schroedinger's time independent equation for all values of x ?* **VTU : Jan.-12**

a) a sin x *b) a cos x*

c) a sec x *d) a sin x + b cos x*

(Ans. : c)

Q.27 *Which of the wave functions shown in Fig. 2.1 cannot be accepted as a solution to Schroedinger's wave equation ?*

(Ans. : d)

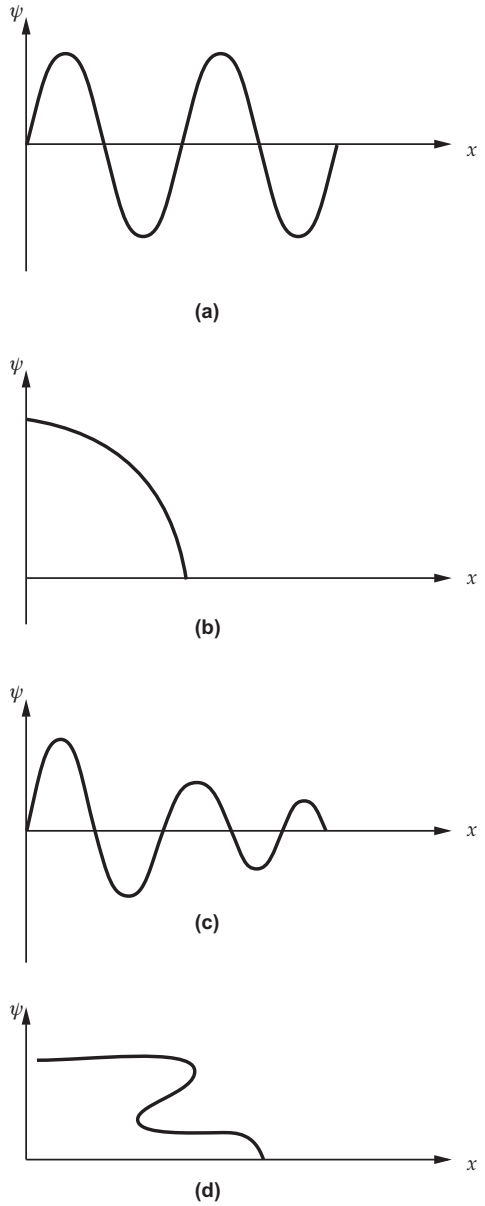


Fig. 2.1

Q.28 *The uncertainty in the determination of position of an electron is* $\left(\frac{h}{3\pi}\right)$. *Then, the uncertainty in the determination of its momentum is* _____ .

VTU : Feb.-11

a) $\frac{3}{4}$ b) $\frac{1}{4}$ c) $\frac{4}{3}$ d) 3

(Ans. : a)

Q.29 *The probabilty of locating a particle is maximum* _____ . **VTU : Feb.-11**

a) at the centre of the wave packet b) at the nodes of the wave packet

c) cannot be determined d) none of these

(Ans. : a)

Q.30 *The ground state energy of an electron in an one dimensional infinite potential well of width 2* $\mathring{A}$ *is 16 eV. Its energy in third excited state is* _____ . **VTU : Feb.-11**

a) 32 eV b) 64 eV c) 144 eV d) 256 eV

(Ans. : d)

Q.31 *To become a nuclear constituent, the K.E. of* $\bar{e}$ *must be of the order of* _____ .

VTU : Aug.-11

a) 20 MeV b) 2 MeV c) 20 eV d) zero

(Ans. : a)

Q.32 *The product of uncertainties between position and momentum is given by* _____.

VTU : Jan.-12

a) $\Delta x \Delta p \geq \lambda$ b) $\Delta x \Delta p \geq \frac{\hbar}{2}$ c) $\Delta x \Delta p \geq mV$ d) $\Delta x \Delta p \geq n\hbar$.

(Ans. : b)

Q.33 *The energy corresponding to the first permitted energy level for a particle in an infinite potential well is called* _____. **VTU : Jan.-12, 13, June-12**

a) excited energy b) zero point energy

c) meta stable state energy d) none of these.

(Ans. : b)

Q.34 *In quantum mechanics the energy operation is represented as :* **VTU : Jan.-13**

a) $\frac{8\pi^2 m \partial^2}{h^2 \partial x^2}$ b) $-\frac{h^2}{4\pi^2 m}\frac{\partial^2}{\partial x^2}$ c) $-\frac{h^2}{8\pi^2 m}\frac{\partial^2}{\partial x^2}$ d) $\frac{h^2}{2\pi^2 m}\frac{\partial^2}{\partial x^2}$

(Ans. : c)

Q.35 *The probability of finding the particle within an element of volume* $d\tau$ *is* _____.

VTU : Jan.-13

a) zero b) $\int |\psi|^2 d\tau$ c) $\int |\psi| d\tau$ d) $\int |\psi^*| d\tau$

(Ans. : b)

Q.36 *The energy of a particle* E_n *in one-dimensional potential box of width L and infinite height is* _____.

VTU : Jan.-13

a) $nh/8mL^2$ b) $nh/8mL$ c) n^2h^2/mL^2 d) $n^2h^2/8mL^2$

(Ans. : d)

Q.37 *The energy of the lowest state in one dimensional potential box of length a = 1 unit is _____.* **VTU : June-13**

a) $\frac{h^2}{8m}$ b) zero c) $\frac{h^2}{4ma^2}$ d) $\frac{h^2}{2ma^2}$

(Ans. : a)

Q.38 *For a particle which is not bound to any system and is free, the energy eigen value is _____.* **VTU : June-13**

a) zero b) finite but not quantized

c) infinity d) finite but quantized

(Ans. : b)

Q.39 *If the uncertainty in the position of a particle is equal to its de-Broglie wavelength then uncertainty in its momentum will be _____.* **VTU : June-13**

a) $\Delta P \geq \frac{h}{4\pi}$ b) $\Delta P \geq \frac{h}{2\pi}$ c) $\Delta P \geq \frac{P}{4\pi}$ d) $\Delta P \geq \frac{h}{P}$

(Ans. : c)

Q.40 *For an electron to be present inside the nucleus of an atom the uncertainty in the position of the electron must be _____.* **VTU : June-13**

a) more than or equal to the radius of the nucleus

b) more than or equal to the diameter of the nucleus

c) more than the diameter of the nucleus

d) less than or equal to the diameter of the nucleus

(Ans. : d)

❑❑❑

Notes

3 Electrical Properties of Materials

Chapter at a Glance

- **Assumptions in classical free electron theory**
  1) The metal atoms have a core, which consists of a nucleus surrounded by the inner electrons and the valence electrons. The positive ion core along with the valence electrons is electrically neutral. The valence electrons are free to move throughout the volume of the sample.
  2) The electrostatic forces of attraction between the free electrons and the ion cores are negligible.
  3) The electrostatic forces of repulsion amongst the free electrons are negligible.
  4) The electrons cannot escape from the metal due to potential barrier at the surface. Inside the metal the potential is constant.
  5) The free electrons are equivalent to gas molecules and hence the kinetic theory of gases is applicable to them. In absence of electric field, they have random thermal motion. Their energies are distributed according to the Maxwell-Boltzmann distribution.

The drift velocity is given by

$$v_d = \frac{eE}{m}\tau$$

- **Expression for Current in terms of Drift Velocity**

$$I = nea v_d$$

- **Expression for Current Density in terms of Conductivity and ElectricField**

$$J = \sigma E$$

- **Expression for Current Density in terms of Relaxation Time**

$$J = \frac{ne^2 E\tau_r}{m}$$

- **Expression for Conductivity in terms of Relaxation Time**

$$\sigma = \frac{ne^2 \tau_r}{m}$$

- **Expression for Resistivity in terms of Relaxation Time**

$$\rho = \frac{1}{\sigma} = \frac{m}{n e^2 \tau_r}$$

- **Mobility of electrons :** The quantity $\frac{v_d}{E}$ is defined as the mobility of electrons.

- **Condutivity in terms of mobility**

$$\sigma = n e \mu$$

- **Temperature Dependence of Resistivity from Classical Free Electron Theory :**

$$\rho = \frac{\sqrt{3 mkT}}{n e^2 \lambda}$$

$$\therefore \quad \rho \propto \sqrt{T}$$

- **Assumptions in quantum free electron theory**
  1) The valence electrons are free to move inside the metal.
  2) The electrons are confined to the metal by potential barrier at the boundaries. The potential is constant inside the metal.
  3) The electrostatic forces of attraction between the free electrons and the ion cores are negligible.
  4) The electrostatic forces of repulsion amongst the free electrons are negligible.
  5) The energies of electrons are quantized and the distribution of electrons in the allowed descrete energy levels is according to Pauli's exclusion principle which prohibits more than one electron in a single quantum state.
- The Fermi-Dirac probability distribution function, also known as fermi function, is

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

where $f(E)$ = Probability of an electron occupying the energy state E

E_F = Fermi energy

k = Boltzmann constant

and T = Absolute temperature

- The fermi energy E_F at temperatures other than 0 K is related to E_{F0} as

$$E_F = E_{F0}\left[1-\frac{\pi^2}{12}\left(\frac{kT}{E_{F0}}\right)^2\right]$$

- The velocity of electrons at the fermi surface, i.e. the electrons having energy equal to the fermi energy, is known as the fermi velocity. It is given by

$$V_F = \sqrt{\frac{2E_F}{m}}$$

- The quantity $\frac{E_F}{k}$, where k is Boltzmann constant is known as the fermi temperature.

$$T_F = \frac{E_F}{k}$$

- According to quantum free electron theory,

$$\rho \propto T$$

- **Merits of Quantum Free Electron Theory**
  1) The quantum free electron theory gives the correct dependence of resistivity on temperature.
  2) It gives the correct molar specific heat at constant volume for the free electrons in a metal which is very low compared to that predicted by classical free electron theory.
- **Comparison between Classical and Quantum Free Electron Theories**

A) Similarities

1) In both models, the conduction is due to free electrons which are the valence electrons.
2) The repulsion between electrons is neglected.
3) The attraction between electrons and positive ion cores is neglected.
4) The electrons are assumed to be confined to the metal by potential barriers at the boundaries and the potential is assumed constant inside the metal.

B) Dissimilarities

Sr. No.	Classical theory	Quantum theory
1.	Electron energy is distributed according to Maxwell-Boltzmann statistics.	Electron energy is distributed according to Fermi-Dirac statistics.
2.	Electrons can have continuous energy values.	Electron energies are discrete and quantized.
3.	Many electrons can have the same energy.	No two electrons can have the same energy.

- **Conductivity of Intrinsic Semiconductor**

$$\sigma = n_i e (\mu_e + \mu_h)$$

- **Conductivity of n-type Semiconductor**

$$\sigma \approx n_e e \mu_e$$

- **Conductivity of p-type Semiconductor**

$$\sigma \approx n_h e \mu_h$$

- **Position of Fermi Level in Intrinsic Semiconductors :** The Fermi level lies exactly at the centre of the forbidden gap.

Hall Effect : When magnetic field is applied perpendicular to direction of current in a conductor, a potential difference develops along an axis perpendicular to both current and magnetic field. This effect is known as Hall effect and the potential difference developed is known as Hall voltage.

- **Hall Voltage :** $V_H = \dfrac{I B d}{n q a} = R_H \dfrac{I B d}{a}$

where R_H is the Hall coefficient.

Important Theory Questions and Answers

Q.1 *Based on free electron theory, derive an expression for electrical conductivity of metals. How does electrical resistance change with impurity and temperature ?*

VTU : Feb.-08, Marks 9

Ans. : In absence of any applied electric field, the electrons have random thermal motion. They keep colliding with ion cores. The direction of velocity changes after every collision. The average velocity of all the electrons in such a situation is zero and there is no net current in the conductor.

In presence of an external applied electric field, the electrons have a small net component of velocity directed opposite to the electric field. The electrons do not move in a straight line opposite to the electric field due to collisions with ion cores. They move in a zig-zag path with a small average velocity opposite to the electric field which is known as the drift velocity. The drift velocity does not depend on time even though the electrons are accelerated between two collisions by the electric field.

The average time between two collisions is known as mean collision time and the average distance covered by the electrons between two collisions is known as mean free path.

When applied electric field is turned off, the drift velocity decreases and falls off to zero exponentially. The time required by the drift velocity to decrease to $\left(\dfrac{1}{e}\right)$ times the initial value is known as the relaxation time.

The expression for drift velocity is

$$v_d = \frac{e E \tau_r}{m} \quad \text{... (1)}$$

where τ_r is the relaxation time.

The amount of charge that flows through any area in $1s$ is defined as the current.

$$\therefore \quad I = n e a v_d \quad \text{... (2)}$$

The current density is defined as the current per unit area

$$\therefore \quad J = \frac{I}{a} = n e v_d \quad \text{... (3)}$$

Resistivity or specific resistance is defined as the resistance of a conductor of unit length and unit area of cross section. It is related to the resistance 'R' by the following expression.

$$R = \rho \frac{l}{a} \quad \text{... (4)}$$

where ρ = Resistivity

l = Length of conductor

and a = Area of cross section

Conductivity is defined as the reciprocal of resistivity. i.e.,

$$\text{Conductivity } \sigma = \frac{1}{\rho}$$

The current density is also given by

$$\therefore \quad J = \sigma E \quad \text{... (5)}$$

From equation (1),

$$v_d = \frac{e E \tau_r}{m}$$

and from equation (3),

$$J = n e v_d$$

$$\therefore \quad J = n e \times \frac{e E \tau_r}{m}$$

$$\therefore \quad J = \frac{n e^2 E \tau_r}{m} \quad \text{... (6)}$$

As $J = \sigma E$,

$$\sigma = \frac{n e^2 \tau_r}{m}$$

Q.2 *Explain fermi energy and fermi factor. Discuss the variation of fermi factor with temperature and energy.* **VTU : Aug.-07, Feb.-10, Marks 8**

Ans. : The Fermi-Dirac probability distribution function, also known as fermi function, is

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}} \quad \ldots (1)$$

where $f(E)$ = Probability of an electron occupying the energy state E

E_F = Fermi energy

k = Boltzmann constant

and T = Absolute temperature

for $T = 0\ K$, if $E > E_F$

$$f(E) = \frac{1}{1+e^{\infty}}$$

$$= \frac{1}{1+\infty}$$

$$\therefore \quad f(E) = 0$$

i.e. no electron can have energy greater than the fermi energy at 0 *K*. It means that all energy states above the fermi energy are empty at 0 *K*.

for $T = 0\ K$, if $E < E_F$,

$$f(E) = \frac{1}{1+e^{-\infty}}$$

$$= \frac{1}{1+0}$$

$$\therefore \quad f(E) = 1$$

i.e. all electrons occupy energy states below the fermi energy at 0 *K*.

Thus, all energy states below fermi energy are filled and energy states above fermi energy are empty at 0 *K*. Hence fermi energy as the highest occupied energy state at 0 *K*.

for $T > 0\ K$, if $E = E_F$,

$$f(E_F) = \frac{1}{1+e^{0}} = \frac{1}{1+1}$$

$$\therefore \quad f(E_F) = \frac{1}{2}$$

i.e. the fermi energy level represents the energy state with a 50 % probability of being filled if forbidden gap does not exist as in the case of good conductors.

The fermi function described by equation (1) is shown for different temperatures in Fig. 3.1.

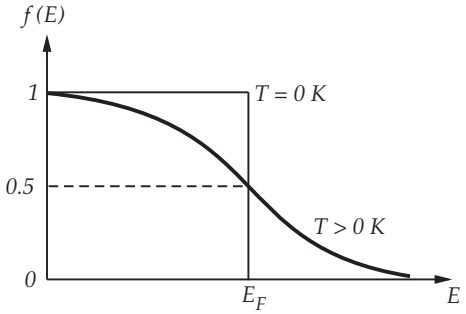


Fig. 3.1

Q.3 *Discuss the various drawbacks of classical free electron theory of metals. What are the assumptions made in quantum theory to overcome the same ?*

VTU : Aug.-09, Marks 8, Aug.-11, Marks 6

Ans. : Drawbacks of Classical Free Electron Theory

1) The classical free electron theory predicts that the resistivity is directly proportional to square root of the absolute temperature. However, experimental evidence shows that the resistivity is directly proportional to the absolute temperature.

2) According to classical theory, the free electron gas should contribute to the molar specific heat at constant volume of metal an amount equal to

$$C_V = \frac{3}{2} R$$

Experiments show that the contribution of free electrons to the molar specific heat at constant volume of any metal is much smaller ($C_V = 10^{-4}\, RT$). Also, the contribution of free electrons is found to be temperature dependent which is not predicted by the classical free electron theory.

3) According to classical theory, the mean free path must be of the order of the interatomic spacing in the crystal. However, calculations from various experiments show that the mean free path is many interatomic spacings and it also depends on temperature.

4) The classical free electron theory predicts larger conductivity for divalent and trivalent atoms like zinc and aluminium compared to monovalent atoms like copper and silver as it is based on the assumption that the valence electrons take part in conduction. This is contrary to observations that copper and silver are better conductors than zinc and aluminium.

5) The classical free electron theory predicts that the magnetic susceptibility of metals to be inversely proportional to temperature whereas experimentally it is established that the susceptibility is independent of temperature.

Assumptions in quantum free electron theory

1) The valence electrons are free to move inside the metal.
2) The electrons are confined to the metal by potential barrier at the boundaries. The potential is constant inside the metal.
3) The electrostatic forces of attraction between the free electrons and the ion cores are negligible.
4) The electrostatic forces of repulsion amongst the free electrons are negligible.
5) The energies of electrons are quantized and the distribution of electrons in the allowed descrete energy levels is according to Pauli's exclusion principle which prohibits more than one electron in a single quantum state.

Important Solved Examples

Example 3.1 *Find the temperature at which there is 1.0 % probability that a state with an energy 0.5 eV above fermi energy will be occupied.*

VTU : Feb.-04, 07, 09, Aug.-08, Marks 5

Solution : The fermi distribution function is

$$f(E) = \frac{1}{1+e^{(E-E_F)/kT}}$$

$$f(E) = 1.0\ \% = 0.01$$

$$E-E_F = 0.5\ eV = 0.5\times 1.6\times 10^{-19}\ J$$

$$\therefore \quad E-E_F = 8\times 10^{-20}\ J$$

$$k = 1.38\times 10^{-23}\ J/K$$

$$\therefore \quad 0.01 = \frac{1}{1+e^{\left(\frac{8\times 10^{-20}}{1.38\times 10^{-23}\ T}\right)}}$$

$$1+e^{\left(\frac{5797.1}{T}\right)} = 100$$

$$e^{\left(\frac{5797.1}{T}\right)} = 99$$

$$\frac{5797.1}{T} = 4.595$$

$$\therefore \quad \boxed{T = 1261.6\ K}$$

Example 3.2 *Calculate the number of acceptors to be added to a Germanium sample to obtain the resistivity of 10 Ω-cm.*
Given : μ = 1700 cm²/V - sec. **PU : May-08, 12**

Solution :

$$\rho = \frac{1}{n_h e \mu_h}$$

$$\therefore \quad n_h = \frac{1}{\rho e \mu_h}$$

$$\rho = 10\ \Omega\text{-cm}$$

$$e = 1.6 \times 10^{-19}\ \text{C}$$

$$\mu_h = 1700\ \text{cm}^2/\text{V - s}$$

$$\therefore \quad n_h = \frac{1}{10 \times 1.6 \times 10^{-19} \times 1700}$$

$$\therefore \quad \boxed{n_h = 3.676 \times 10^{14}\ \text{per cm}^3}$$

Important Multiple Choice Questions

Q.1 *If the mobility of electron in a metal increases, the resistivity _____.*

a) decreases *b) increases* *c) remains constant* *d) none of these.*

VTU : Jan.-09, 12, Aug.-11
(Ans. : a)

Q.2 *Ohm's law relates to the electric field E, conductivity σ and current density J as _____.*

a) $J = \frac{E}{\sigma}$ *b)* $J = \sigma E^2$ *c)* $J = \frac{\sigma}{E}$ *d)* $J = \sigma E$

VTU : Feb.-09, June-13
(Ans. : d)

Q.3 *The average drift velocity v_d of electrons in a metal is related to the electric field E and collision time τ as _____.*

a) $\sqrt{\frac{eE\tau}{m}}$ *b)* $\sqrt{\frac{m}{eE\tau}}$ *c)* $\frac{eE\tau}{m}$ *d)* $\frac{m}{eE\tau}$

VTU : Feb.-09
(Ans. : c)

Q.4 *Experimentally specific heat at constant volume c_V is given by _____.*

a) $\frac{3}{2}R$ *b)* $10^{-4}Rt$ *c)* $\frac{2}{3}R$ *d)* $10^{-4}R$

VTU : Feb.-09
(Ans. : b)

Q.5 *If the mobility of electron is 7×10^{-3} m^2/V-s when accelerated by a field of 1 V/cm, then v_d is _____.*

a) 7×10^{-3} m/s b) 0.7 m/s c) 7×10^{-2} m/s d) 0.007 m/s

VTU : Aug.-09

(Ans. : b)

Q.6 *The temperature dependence for electrical resistivity of metal is _____.*

a) $\rho \propto \frac{1}{T}$ b) $\rho \propto \frac{1}{\sqrt{T}}$ c) $\rho \propto \sqrt{T}$ d) $\rho \propto T$

VTU : Aug.-09, 11

(Ans. : d)

Q.7 *The fermi factor for $E = E_F$ at $T > 0$ K is _____.*

a) 1 b) $\frac{1}{2}$ c) 0 d) 2

VTU : Aug.-09

(Ans. : b)

Q.8 *According to quantum free electron theory, the energy levels in a metal are _____.*

a) continuous b) discrete c) overlapping d) none

VTU : Aug.-09

(Ans. : b)

Q.9 *For ordinary metals, the resistivity versus temperature curve at T = 0, _____.*

a) has a positive intercept b) has a negative intercept

c) goes through the origin d) none of these

VTU : Jan.-10, 12

(Ans. : a)

Q.10 *Which of the following relations is correct for current density ?*

a) $J = n e A v_d$ b) $J = \frac{n e A}{v_d}$ c) $J = n e v_d$ d) $J = \frac{1}{n e v_d}$

VTU : Feb.-10

(Ans. : c)

Q.11 *The value of fermi distribution function at absolute zero (T = 0 K) is 1 under the condition _____.*

a) $E = E_F$ b) $E > E_F$ c) $E >> E_F$ d) $E < E_F$

VTU : Feb.-10

(Ans. : d)

Q.12 *If the fermi energy of silver is 5.5 eV, the fermi velocity of conduction electrons is _____.*

a) 0.98×10^6 m/s b) 1.39×10^6 m/s c) 2.46×10^5 m/s d) none of these.

VTU : Feb.-10

(Ans. : b)

$$\frac{1}{2} m v_F^2 = E_F$$

$$\therefore V_F = \sqrt{\frac{2 \times 5.5 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$$\therefore V_F = 1.39 \times 10^6 \ m/s$$

Q.13 *The value of fermi distribution function at absolute zero is 0 under the condition _____.*

a) $E = E_F$ *b)* $E > E_F$ *c)* $E >> E_F$ *d)* $E < E_F$

(Ans. : b)

Q.14 *The value of fermi factor for* $E = E_F$ *at any temperature other than 0 K is _____.*

a) 0 *b) 1* *c) 0.5* *d) 0.75*

(Ans. : c)

Q.15 *The electron energies in classical free electron theory follow _____.*

a) Maxwell-Boltzmann statistics *b) Fermi-Dirac statistics*

c) Bose-Einstein statistics *d) none of the above*

(Ans. : a)

Q.16 *The electron energies in quantum free electron theory follow _____.*

a) Maxwell-Boltzmann statistics *b) Fermi-Dirac statistics*

c) Bose-Einstein statistics *d) none of the above*

(Ans. : b)

Q.17 *According to classical free electron theory, the resistivity is proportional to _____.*

a) T *b)* $\frac{1}{T}$ *c)* $\sqrt{T}$ *d)* $\frac{1}{\sqrt{T}}$

(Ans. : c)

Q.18 *The resistivity of metals is due to scattering of electrons by _____.*

a) phonons *b) lattice imperfections*

c) impurities *d) all the above*

(Ans. : d)

Q.19 *The resistivity in metals due to impurities and lattice imperfections _____.*

a) increases with temperature *b) decreases with temperature*

c) remains approximately constant at all temperatures

d) initially increases and then decreases with temperature

(Ans. : c)

Q.20 *The resistivity in metals due to scattering by phonons _____.*

a) increases with temperature *b) decreases with temperature*

c) remains independent of temperature *d) none of the above*

(Ans. : a)

Q.21 *If x is the probability of occupancy of an energy level* ΔE *below the fermi level, then the probability of non-occupancy at an energy level* ΔE *above the fermi level is _____.*

a) x *b) 0* *c) 1* *d) 0.5*

(Ans. : a)

Q.22 *The fermi temperature is given by _____.*

a) $\frac{2}{3}\frac{E_F}{k}$ *b)* $\frac{3}{2}\frac{E_F}{k}$ *c)* $\frac{E_F}{k}$ *d)* $\frac{2E_F}{k}$

(Ans. : c)

Q.23 *The potential inside the metal is assumed to be constant in _____.*

a) only classical theory *b) only quantum theory*

c) both classical and quantum theories *d) neither classical nor quantum theory*

(Ans. : c)

Q.24 *Many electrons can have the same energy in _____.*

a) only classical theory *b) only quantum theory*

c) both classical and quantum theories *d) neither classical nor quantum theory*

(Ans. : a)

Q.25 *Electrons cannot have energy greater than the fermi energy at temperature of _____.*

a) 0 K *b) 273.16 K* *c) 300 K* *d) 325 K*

(Ans. : a)

Q.26 *The mobility of electrons in a conductor is* $4\times10^{-3}\ m^2V^{-1}s^{-1}$. *Then the drift velocity of the electron in the presence of applied electric field of strength 100* Vm^{-1} *is _____.*

a) $4\ ms^{-1}$ *b)* $10\ ms^{-1}$ *c)* $0.4\ ms^{-1}$ *d)* $0.04\ ms^{-1}$

VTU : Feb.-11

(Ans. : c)

Q.27 *The classical value of molar specific heat of a conductor is _____.*

a) $\frac{3}{2}R$ *b)* $\frac{1}{2}R$ *c)* $3\ R$ *d)* $\frac{5}{2}R$

VTU : Feb.-11

(Ans. : a)

Q.28 *The fermi energy of a metal at absolute zero temperature is proportional to _____.*

a) $n^{1/3}$ *b)* $n^{3/2}$ *c)* $n^{2/3}$ *d)* n^2

VTU : Feb.-11

where 'n' is number of free electrons per unit volume.

(Ans. : c)

Q.29 *At 50 K, the probability of finding an electron at fermi energy is* 1/2 . *The probability of finding electron at the same energy level at 100 K is _____.*

a) 1 *b) zero* *c)* 1/4 *d)* 1/2

VTU : Feb.-11

(Ans. : d)

Q.30 *Zero percentage probability is the probability for* $\bar{e}$ *to occupy the energy level above the Fermi energy level at T = 0K is _____.*

a) $E+E_F$ *b)* $E = E_F$ *c)* $E > E_F$ *d)* $E< E_F$

VTU : Aug.-11

(Ans. : c)

Q.31 *If the fermi energy of a metal is 1.4 eV, the fermi temperature of the metal is approximately _____.*

a) 1.6×10^3 K *b)* 1.6×10^4 K *c)* 1.6×10^5 K *d)* 1.6×10^6 K

VTU : Aug.-11

(Ans. : b)

Q.32 *At T > 0 K, the probability of occupancy of Fermi level is _____.*

a) 75 % *b) 90 %* *c) 100 %* *d) 50 %*

VTU : Jan.-12, June-13

(Ans. : d)

Q.33 *The dependence of mean free path λ on temperature T is _____.*

a) $\lambda \alpha T$ *b)* $\lambda \alpha \sqrt{T}$ *c)* $\lambda \alpha \frac{1}{T}$ *d)* $\lambda \alpha \frac{1}{\sqrt{T}}$

VTU : Jan.-12, June-13

(Ans. : c)

Q.34 *In classical free electron theory, the electric field due to ion cores _____.*

a) is neglected *b) is assumed to be periodic*

c) is assumed to be constant *d) None of these*

VTU : June-12

(Ans. : a)

Q.35 *Mobility of electron is _____.*

a) reciprocal of electrical conductivity

b) acceleration of electron per unit electric field

c) average drift velocity per unit electric field

d) None of these

VTU : June-12

(Ans. : c)

Q.36 *If* E_F *is the Fermi energy at absolute zero, then mean energy of the electron at absolute zero is _____.*

a) $\overline{E} = 1.5\ E_F$ *b)* $\overline{E} = \frac{2}{3}E_F$ *c)* $\overline{E} = \frac{2}{5}E_F$ *d)* $\overline{E} = \frac{3}{5}E_F$

VTU : June-12

(Ans. : d)

Q.37 *The resistivity of metals is due to scattering of electron by _____.*

a) phonons *b) lattice imperfection*

c) impurities *d) All of these*

VTU : June-12

(Ans. : d)

Q.38 *The free electrons in classical free electron theory are treated as _____.*

a) rigidly fixed lattice points *b) liquid molecules*

c) gas molecules *d) none of these*

VTU : Jan.-13, June-13

(Ans. : c)

Q.39 *The temperature dependence of classical expression for electrical resistivity of a metal is, _____.*

a) $\rho \propto T^{1/2}$ *b)* $\rho \propto T^{2}$ *c)* $\rho \propto 1/T^{2}$ *d)* $\rho \propto 1/T$

VTU : Jan.-13
(Ans. : a)

Q.40 *The value of Fermi function in Fermi-level is _____ at $T \neq 0$ K.*

a) zero *b) 0.5* *c) 0.75* *d) 1* **VTU : Jan.-13**
(Ans. : b)

Q.41 *If E_F is the Fermi energy at absolute zero, then mean energy $\bar{E}$ of electron at absolute zero is, _____.*

a) $\bar{E} = 1.5\ E_F$ *b)* $\bar{E} = 2/3\ E_F$ *c)* $\bar{E} = 2/5\ E_F$ *d)* $\bar{E} = 3/5\ E_F$

VTU : Jan.-13
(Ans. : d)

Q.42 *The expression for de-Broglie wavelength for an electron under an accelerating potential V is _____.*

a) $\frac{12.26}{\sqrt{V}}\ m$ *b)* $\frac{12.26}{\sqrt{V}}\ \mathring{A}$ *c)* $\frac{12.26}{\sqrt{V}}\ nm$ *d)* $\frac{12.26}{\sqrt{V}}\ \mu m$

VTU : June-13
(Ans. : b)

Q.43 *A particle moves with velocity $3 \times 10^{6}\ ms^{-1}$. The wavelength associated with it is 1 nm. Then group velocity of the particle is _____.*

a) $3 \times 10^{8}\ ms^{-1}$ *b)* $3 \times 10^{10}\ ms^{-1}$ *c)* $3 \times 10^{6}\ ms^{-1}$ *d)* $1.5 \times 10^{6}\ ms^{-1}$

VTU : June-13
(Ans. : c)

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4 Quantum Mechanics

Chapter at a Glance

- For certain materials, like mercury, the resistivity suddenly drops to zero at very low temperature typically near the boiling point of liquid helium. This phenomenon is known as superconductivity.
- In superconducting materials, the resistivity suddenly drops to zero at a particular temperature known as critical temperature (T_C) and remains zero below that temperature.
- When a specimen is placed in a weak magnetic field and cooled below the critical temperature, the magnetic flux originally present in the specimen is ejected from the specimen. This effect is known as **Meissner effect**.
- Superconductivity is destroyed by sufficiently strong magnetic fields. The minimum value of the applied magnetic field required to destroy superconductivity is called the **critical field** H_c and is a function of temperature.
- When a superconducting ring is placed in a magnetic field and the field is switched off, a current is induced in the ring. The magnitude of current remains constant eventhough there is no source of e.m.f. as the resistance of the superconducting ring is zero. Such steady currents flowing in a superconductor are called **persistent currents**.
- According to observations, the critical temperature of superconductors varies with isotopic mass. The critical temperature is smaller for larger isotopic mass. This phenomenon is known as **isotope effect**.
- Superconductivity was explained by Bardeen, Cooper and Schrieffer on the basis of quantum theory. This theory which is named after the three scientists is known as the **BCS theory**.
- There are two types of superconductors based on the difference in magnetization exhibited by them. These are known as Type-I (or soft) and Type-II (or hard) superconductors.
- Superconductors are used in superconducting magnets, magnetic levitation trains and superconducting quantum interference devices.

Important Theory Questions and Answers

Q.1 *Discuss BCS theory of superconductivity.* **VTU : Aug.-08, Jan.-10, June-10, Marks 6**

Ans. : When a free electron passes near an ion in a lattice, there is coulomb attraction between them. This causes deformation of the lattice.

A second free electron passing nearby, takes advantage of the deformation to lower its energy. Thus the second electron interacts attractively with the first through the lattice deformation.

This interaction is called electron - lattice - electron interaction which can be considered to take place through phonons. The quantum of lattice vibrational energy is called phonon, similar to quantum of electromagnetic which energy which is called photon.

The first electron emits a phonon which is absorbed by the second electron. Electrons in such an interaction have same energy and equal and opposite spins. Such pairs of electrons coupled together through a phonon are called **Cooper pairs**.

The attraction between electrons in a Cooper pair is very weak and the pair can be separated by a small increase in temperature which causes thermal vibrations.

At very low temperature, density of Cooper pairs is large and they move collectively through the lattice which minimizes collisions. Also, due to large density of Cooper pairs, their velocity is small even for large currents. Due to small velocity and the collective movement of the Cooper pairs, collisions are reduced and resistivity decreases.

The BCS theory was able to explain the critical field, the isotope effect and the Meissner effect. It could also explain the quantization of magnetic field in a superconducting ring with effective charge '2*e*' instead of '*e*' as the ground state in BCS theory consists of pairs of electrons.

Q.2 *What is superconductivity ? Describe type-I and type-II superconductors.* **VTU : Feb.-08, June-13, Marks 8**

Ans. : For certain materials, like mercury, the resistivity suddenly drops to zero at very low temperature typically near the boiling point of liquid helium. This phenomenon is known as superconductivity.

Type-I Superconductors (Soft superconductors)

The magnetization curve for a Type-I superconductor is shown in Fig. 4.1. Type-I superconductors show complete Meissner effect when the applied magnetic field H is less than the critical field H_c.

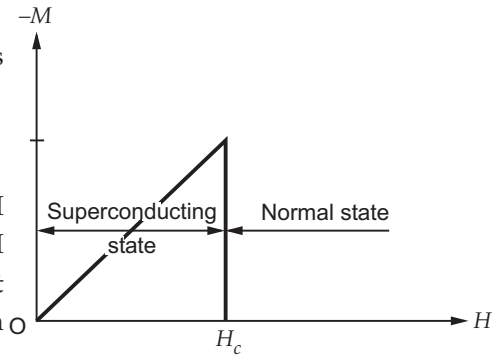

Fig. 4.1

The magnetization in the specimen is proportional to the applied field and the specimen acts like a perfect diamagnet.

For $H > H_c$, the magnetization is negligible and the specimen becomes a normal conductor. Pure elements show such magnetization curves. The values of H_c are very low.

Type-II Superconductors (Hard superconductors)

The magnetization curve for a type-II superconductor is shown in Fig. 4.2.

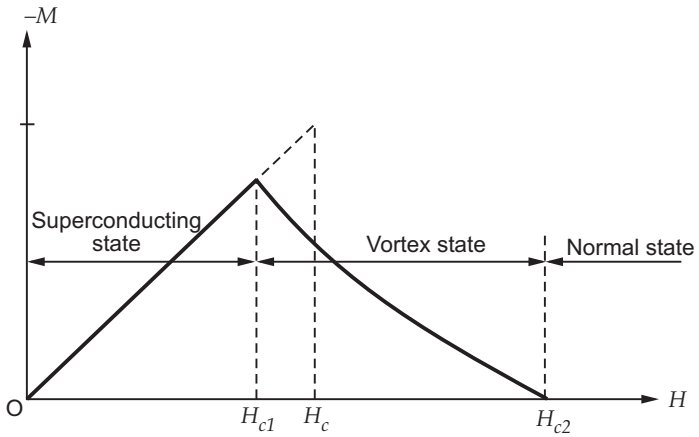

Fig. 4.2

Type-II superconductors are characterized by two critical fields H_{c1} and H_{c2}. Upto the lower critical field H_{c1}, the specimen is perfectly diamagnetic and flux is completely ejected out from the specimen. The magnetization in the specimen is proportional to the applied field. Between H_{c1} and H_{c2}, Meissner effect is incomplete but electrically the specimen is a superconductor. This state is called the mixed state or the vortex state as the specimen is a electrically a superconductor but not magnetically.

The values of H_{c2} may be 100 times or more higher than the critical field H_c for type-I superconductors.

Above the higher critical field H_{c2}, the specimen becomes a normal conductor. Type-II superconductors are usually alloys or transition metals with high values of electrical resistivity in the normal state.

Important Multiple Choice Questions

Q.1 *The temperature at which superconductivity occurs is called _____.*

a) low temperature *b) super temperature*

c) critical temperature *d) high temperature* **VTU : Feb.-09**

(Ans. : c)

Q.2 *Superconductivity phenomenon can be explained on the basis of _____.*

a) BCS theory *b) CCS theory* *c) DCS theory* *d) MCS theory*

VTU : Feb.-09

(Ans. : a)

Q.3 *Meissner effect will take place in _____.*

a) solid *b) superconducting magnet*

c) Maglev vehicle *d) MRI*

VTU : Feb.-09

(Ans. : c)

Q.4 *The width of the energy gap of a superconductor is maximum at _____.*

a) T_c *b) 0 K* *c)* $\frac{T_c}{2}$ *d)* $\frac{T_c}{3}$

VTU : Aug.-09

(Ans. : b)

Q.5 *When the type-I superconducting material is placed in an external magnetic field, it _____.*

a) attracts the magnetic lines *b) enhances the magnetic field lines*

c) repels the magnetic field lines *d) does not influence magnetic field lines*

(Ans. : c)

Q.6 *The superconductor behaves like a perfect _____.*

a) paramagent *b) ferromagnet* *c) diamagnet* *d) ferrimagnet*

VTU : Jan.-12

(Ans. : c)

Q.7 *Magnetic field due to a superconducting ring is quantized with effective charge _____.*

a) e *b) 2e* *c) 3e* *d) zero*

(Ans. : b)

Q.8 *The number of critical fields in a type-I superconductor is _____.*

a) 0 *b) 1* *c) 2* *d) 3*

(Ans. : b)

Q.9 *The number of critical fields in a type-II superconductor is _____.*

a) 0 *b) 1* *c) 2* *d) 3*

(Ans. : c)

Q.10 *A type-II superconductor in the vortex state shows _____.*

a) complete meissner effect and zero electrical resistivity

b) incomplete meissner effect and zero electrical resistivity

c) complete meissner effect and non-zero electrical resistivity

d) incomplete meissner effect and non-zero electrical resistivity

(Ans. : b)

Q.11 *At temperatures below critical temperature, if a superconductor is subjected to magnetic field greater than critical field H_c, it _____.*

a) remains superconducting *b) changes to normal state*

c) does not expel the magnetic field lines but has zero resistivity

d) none of these

(Ans. : b)

Q.12 *Below critical temperature, if the temperature of superconductor is increased, the critical field _____.*

a) increases *b) decreases* *c) remains constant* *d) none of the above*

VTU : Jan.-12

(Ans. : b)

Q.13 *The magnetic susceptibility of a superconductor is _____.*

a) 0 *b) 1* *c) – 1* *d) 0.5*

(Ans. : c)

Q.14 *The variation of critical field H_c with temperature T is given by _____.*

a) $H_c = H_0\left[1-\left(\frac{T}{T_c}\right)^2\right]$ *b)* $H_c = H_0\left[1+\left(\frac{T}{T_c}\right)^2\right]$

c) $H_c = H_0\left[1-\frac{T}{T_c}\right]$ *d)* $H_c = H_0\left[1+\frac{T}{T_c}\right]$

(Ans. : a)

Q.15 *If a bar magnet is released from a point above a superconducting coil, the magnet will _____.*

a) accelerate downward *b) move upwards*

c) float in air *d) none of the above.*

(Ans. : c)

Q.16 *The critical temperature of mercury is _____.*

a) 4.2 K *b) 6.2 K* *c) 7.8 K* *d) 20 K*

VTU : Feb.-11

(Ans. : a)

Q.17 *The temperature of a superconductor kept in a weak magnetic field is reduced below critical temperature, then _____.*

a) $R = 0; B \neq 0$ *b)* $R \neq 0; B = 0$ *c)* $R \neq 0; B \neq 0$ *d)* $R = 0; B = 0$

VTU : Feb.-11

(Ans. : d)

Q.18 *The conductivity of a superconductor is _____.*

a) infinite *b) zero* *c) finite* *d) none of these*

VTU : Aug.-11

(Ans. : a)

Q.19 *The relation between superconducting transition temperature (T_C) and atomic weight (μ) of isotope is _____.*

a) $T_C \propto \mu$ *b)* $T_C \propto \frac{1}{\mu}$ *c)* $T_C \propto \sqrt{\mu}$ *d)* $T_C \propto \frac{1}{\sqrt{\mu}}$

VTU : Aug.-11
(Ans. : d)

Q.20 *In a superconductor in superconducting state critical magnetic field _____.*

a) increases if temperature decreases *b) increase with increase in temperature*

c) does not depend on temperature *d) remain constant*

VTU : June-12
(Ans. : a)

Q.21 *According to BCS theory, the Cooper pair is pair of _____.*

a) electron-proton *b) electron-electron* *c) proton-proton* *d) electron-neutron*

VTU : Jan.-13
(Ans. : b)

Q.22 *High temperature superconductors bear the crystal structure of _____.*

VTU : Jan.-13

a) cubic *b) orthorhombic* *c) diamond* *d) perovskite*

(Ans. : d)

Q.23 *According to Meissner effect, material in superconducting state is _____.*

a) paramagnetic *b) diamagnetic* *c) ferromagnetic* *d) anti-ferromagnetic*

VTU : Jan.-13
(Ans. : b)

Q.24 *Superconductors are _____.*

a) Ferromagnetic *b) Paramagnetic* *c) Antiferromagnetic* *d) Diamagnetic*

VTU : June-13
(Ans. : d)

Q.25 *All high temperature superconductors are different types of oxides of _____.*

a) mercury *b) lead* *c) copper* *d) tin*

VTU : June-13
(Ans. : c)

Q.26 *The quantum of magnetic flux is given by, _____.*

a) $\frac{2e}{h}$ *b)* $\frac{h}{2e}$ *c)* $\frac{he}{2}$ *d)* $\frac{2h}{e}$

VTU : June-13
(Ans. : b)

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5 Lasers

Chapter at a Glance

- The term laser is an acronym for light amplification by stimulated emission of radiation.
- If the phase difference between two points along any ray remains constant, the coherence is called temporal coherence.
- If the phase difference between two points in a plane normal to the ray direction remains constant the coherence is called spatial or lateral coherence.
- **Metastable states :** These are excited states of an atom with relatively larger life times of the order of 10^{-3}s. As these energy states are neither as stable as ground state nor as unstable as the other excited states, they are known as metastable states.
- **Spontaneous emission of radiation :** When an atom in its excited energy state E_2 makes a transition to the ground state E_1 on its own, without any external stimulus, it emits a photon of energy $E_2 - E_1$. This is known as spontaneous emission of radiation.
- **Stimulated emission of radiation :** If a photon having energy $E_2 - E_1$ interacts with an atom in the energy state E_2, the photon forces the atom to undergo transition to the ground state E_1 giving rise to another photon of the same energy $E_2 - E_1$. This is known as stimulated emission of radiation.
- The light produced by stimulated emission is coherent.
- The rate of absorption of photons is proportional to the number density N_1 of the atoms in ground state and the energy density E_v in the frequency range ν to $\nu + d\nu$ in incident radiation,i.e,

Rate of absorption = $B_{12} N_1 E_\nu$

Where B_{12} is a constant known as **Einstein's coefficient of induced absorption**.

- Rate of spontaneous emission = $A_{21} N_2$

where A_{21} is a constant known as **Einstein's coefficient of spontaneous emission** and N_2 is the number of atoms in the excited state.

- Rate of stimulated emission = $B_{21} N_2 E_\nu$

where B_{21} is a constant known as **Einstein's coefficient of stimulated emission**.

- The state, in which there is a larger number of atoms in the higher energy state than the lower, is called **population inversion**. Population inversion ensures amplification of light.
- The process of raising the atoms from a lower energy state to higher, to create population inversion, is called **pumping**.
- Cavities can be constructed using different types of mirrors such that the light rays return to their original location and orientation after travelling through the cavity for a certain number of times. Such cavities are known as **resonant cavities**.
- For a laser cavity of length L and wavelength λ, the number of modes n is given by

$$n = \frac{2L}{\lambda}$$

- The carbon dioxide laser makes use of transitions in the molecular vibrational and rotational energy levels. Carbon dioxide is the active gas having metastable states.
- In a semiconductor laser, the large forward bias applied to the junction is the pumping mechanism which produces population inversion.
- **Characteristics of laser :** Laser is monochromatic, coherent, unidirectional, bright and can be focussed easily.
- Lasers are used for welding, cutting, drilling, measuring atmospheric pollutants, surgeries, telecommunication and holography.
- Holography is a technique of producing three dimensional images.
- For recording a hologram, coherent light from a laser is partly reflected from a mirror (known as reference beam) and partly reflected from the object (known as object beam). The interference pattern is recorded on a photographic plate.
- For reconstructing the image, the hologram is illuminated by reference beam.

Important Theory Questions and Answers

Q.1 *Obtain an expression for energy density of radiation under equilibrium condition in terms of Einstein's coefficients.* **VTU : Jan.-07, 09, 10, 13, Marks 10**

Ans. : Consider a system of atoms having a ground state energy E_1 and excited state energy E_2 with number densities of atoms in these states N_1 and N_2 respectively. If photons of frequency,

$$\nu = \frac{E_2 - E_1}{h}$$

are incident on the system of atoms there will be induced absorption. The rate of absorption of photons will be proportional to the number density N_1 of the atoms in ground state and the energy density E_v in the frequency range ν to $\nu + d\nu$ in incident radiation.

$\therefore$ Rate of absorption $\propto N_1 E_\nu$

$\therefore$ Rate of absorption $= B_{12} N_1 E_\nu$

Where B_{12} is a constant known as Einstein's coefficient of induced absorption.

Atoms in excited state E_2 can come down to ground state either through spontaneous emission or through stimulated emission of radiation. In the case of spontaneous emission, the rate of transition of atoms from E_2 to E_1 does not depend on the energy density in the incident radiation and is proportional only to the number density of atoms in the excited state i.e.,

Rate of spontaneous emission $\propto N_2$

$\therefore$ Rate of spontaneous emission $= A_{21} N_2$

where A_{21} is a constant known as Einstein's coefficient of spontaneous emission.

In the case of stimulated emission, a photon of frequenc $\nu = \dfrac{E_2 - E_1}{h}$ is required to stimulate the atoms. Hence the rate of stimulated emission is proportional to the energy density E_ν and the number density N_2 of the atoms in the excited energy state E_2 i.e.,

Rate of stimulated emission $\propto N_2 E_\nu$

$\therefore$ Rate of stimulated emission $= B_{21} N_2 E_\nu$

where B_{21} is a constant known as Einstein's coefficient of stimulated emission.

In a state of thermal equilibrium, the rate of transition of atoms from E_1 to E_2 must equal the total rate of transition from E_2 to E_1.

$\therefore$ Rate of absorption = Rate of spontaneous emission + Rate of stimulated emission

$$\therefore \quad B_{12} N_1 E_\nu = A_{21} N_2 + B_{21} N_2 E_\nu$$

Dividing by N_1, we get,

$$B_{12} E_\nu = A_{21} \frac{N_2}{N_1} + B_{21} \frac{N_2}{N_1} E_\nu$$

$$\therefore \quad E_\nu \left(B_{12} - B_{21} \frac{N_2}{N_1} \right) = A_{21} \frac{N_2}{N_1}$$

$$E_\nu = \frac{A_{21}\left(\frac{N_2}{N_1}\right)}{B_{12} - B_{21}\left(\frac{N_2}{N_1}\right)}$$

$$E_\nu = \frac{A_{21}}{B_{12}\left(\frac{N_1}{N_2}\right) - B_{21}}$$

$$\therefore \quad E_\nu = \frac{A_{21}}{B_{21}}\left[\frac{1}{\left(\frac{B_{12}}{B_{21}}\right)\left(\frac{N_1}{N_2}\right) - 1}\right]$$

From Maxwell-Boltzman distribution,

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/kT} = e^{-h\nu/kT}$$

$$\therefore \quad \frac{N_1}{N_2} = e^{h\nu/kT}$$

$$\therefore \quad E_\nu = \frac{A_{21}}{B_{21}}\left[\frac{1}{\left(\frac{B_{12}}{B_{21}}\right) e^{h\nu/kT} - 1}\right]$$

Q.2 *Describe the construction and working of semiconductor laser.*

VTU : Feb.-09, Marks 8, Aug.-10, Marks 6

Ans. : Construction : Metallic contacts are provided to the *P* and *N* types in a heavily doped *P-N* junction diode. Two opposite faces which are perpendicular to the plane of the junction are polished and made parallel to each other. These parallel faces constitute the resonant cavity and laser is obtained through these faces as shown in Fig. 5.1. The remaining two faces are roughened to prevent lasing action in that direction.

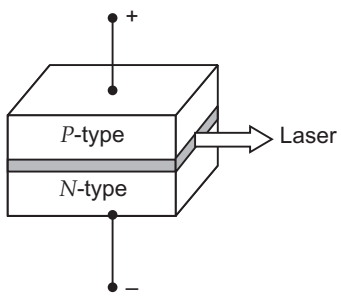

Fig. 5.1

Working : As the *P* and *N* types are heavily doped, the Fermi level (E_F) in *N* type lies in the conduction band and in *P*-type it lies in the valence band. The Fermi level is uniform throughout the unbiased diode as shown in Fig. 5.2 (a).

When the junction is forward biased, the energy levels shift as shown in Fig. 5.2 (b). The width of depletion region decreases due to injection of electrons and holes.

At low forward currents, the electron-hole recombination causes spontaneous emission of radiation and the diode acts as a LED.

When current is increased and reaches a threshold value, population inversion is achieved in the depletion region due to large concentration of electrons in conduction band and holes(i.e. valancies) in valence band. The narrow region where population inversion is achieved becomes the active region where lasing action takes place. The forward bias applied to the junction is thus the pumping mechanism which produces population inversion.

The photons travelling in the junction along the resonant cavity stimulate recombination of electron-hole pairs due to which the intensity of coherent light builds up along the axis of the cavity.

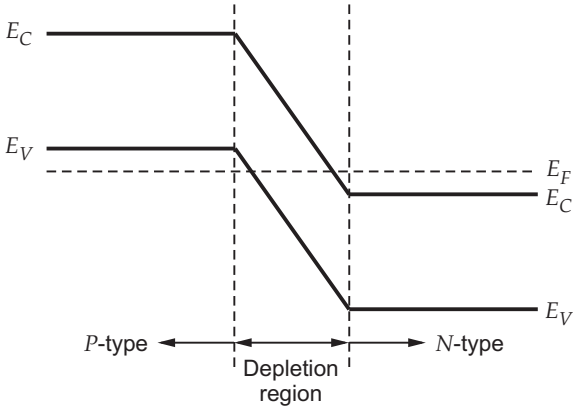

Fig. 5.2 (a)

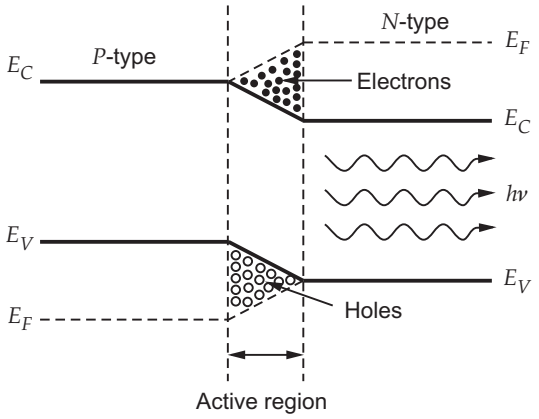

Fig. 5.2 (b)

Q.3 *Write a note on measurement of pollutants in atmosphere using laser.*

VTU : Feb.-07, Aug.-09, Marks 6

Ans. : Pollution in the atmosphere is due to suspended particulate matter like dust, smoke, flash, aerosols etc. and non-particulate matter like carbon monoxide, sulphur dioxide etc. Conventional method for measurement of pollution levels require collection of sample and then its chemical analysis. This is a time consuming process and does not give real time data. Using lasers, we can get real time data by transmitting the laser beam in atmosphere and then observing either the reflected or transmitted beam.

LIDAR (Light detection and ranging) is used to measure concentration of suspended particulate matter in the atmosphere. A pulse of laser is transmitted into the atmosphere and the back-scattered light is recorded using photodiodes. This gives information regarding concentration of suspended particulate matter.

Absorption of certain wavelengths by some gases is used to study their presence in the atmosphere by transmitting a lasers beam through the atmosphere and recording the transmitted intensity.

The presence of different gaseous pollutants is best studied by means of Raman backscattering. The backscattered laser contains the transmitted wavelength as well as longer and shorter wavelengths than the transmitted wavelength. The change in wavelength from the original transmitted wavelength is knows as Raman shift and is specific for particular gas. Hence these gases can be detected.

Q.4 *Describe the recording and reconstruction process in holography with the help of suitable diagrams.* **VTU : Feb.-08, 10, Marks 8**

Ans. : **Recording :** Coherent light from a laser is partly reflected from a mirror (known as reference beam) and partly reflected from the object (known as object beam). The interference pattern of these two beams is recorded on a photographic plate as shown in Fig. 5.3. The developed photographic plate is known as hologram. It consists of alternate bright and dark fringes.

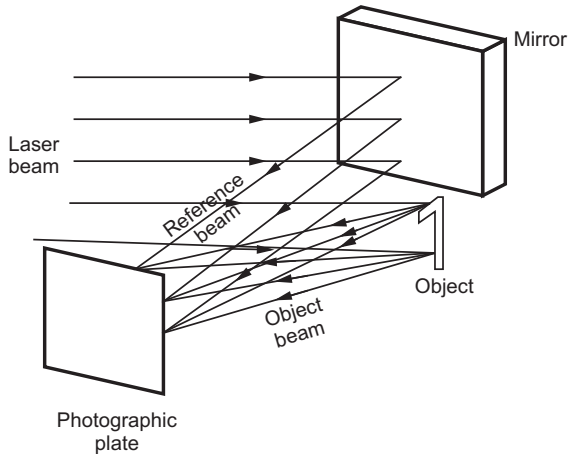

Fig. 5.3

Reconstruction : To see the three dimensional image of the object, the hologram is illuminated by the reference beam only or a laser beam with the orientation of the reference beam with respect to the hologram. The hologram acts as a diffraction grating.

One of the first order diffracted beam produces a real three dimensional and inverted image and the other first order diffracted beam produces a virtual three dimensional image which is formed at the same place where the object was kept during recording as shown in Fig. 5.4.

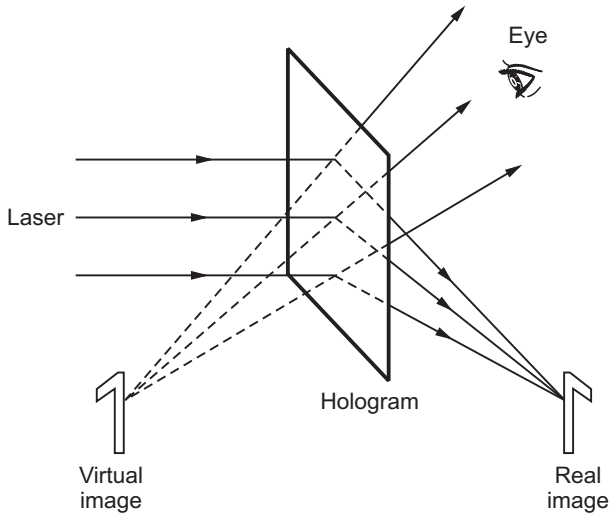

Fig. 5.4

Important Solved Examples

Example 5.1 *Find the number of modes of the standing waves and their frequency separation in the resonant cavity of length 1 m of He-Ne laser operating at wavelength 632.8 nm.*

VTU : Aug.-07, Marks 4, Aug.-11, Marks 5

Solution : For a laser cavity of length L and wavelength λ,

$$L = \frac{n\lambda}{2}$$

$$\therefore \quad n = \frac{2L}{\lambda}$$

$$L = 1\ m$$

$$\lambda = 632.8\ nm = 632.8 \times 10^{-9} m$$

$$\therefore \quad n = \frac{2 \times 1}{632.8 \times 10^{-9}}$$

$$\therefore \quad \boxed{n = 3160556}$$

Frequency separation in cavity is,

$$\Delta \nu = \frac{v}{2L}$$

For gas laser like He-Ne, $v \approx c$

$$\therefore \quad \Delta \nu = \frac{c}{2L}$$

$$= \frac{3 \times 10^{8}}{2 \times 1}$$

$$\therefore \quad \boxed{\Delta \nu = 1.5 \times 10^{8}\ Hz}$$

Example 5.2 *A He-Ne laser is emitting a beam with an average power of 4.5 mW. Find the number of photons emitted per second by the laser. The wavelength of the emitted radiations is 6328 Å.*

VTU : Jan.-08, 12, Marks 4

Solution : $E = P \times t = \frac{Nhc}{\lambda}$

$$\therefore \quad N = \frac{P\,t\lambda}{h\,c}$$

$$P = 4.5\ mW = 4.5 \times 10^{-3}\ W$$

$$t = 1\ s$$

$$\lambda = 6328\ \mathring{A} = 6328 \times 10^{-10}\ m$$

$$h = 6.63 \times 10^{-34}\ J\text{-}s$$

$$c = 3 \times 10^{8}\ m/s$$

$$\therefore \quad N = \frac{4.5 \times 10^{-3} \times 1 \times 6328 \times 10^{-10}}{6.63 \times 10^{-34} \times 3 \times 10^{8}}$$

$$\therefore \quad \boxed{N = 1.432 \times 10^{16}}$$

Important Multiple Choice Questions

Q.1 *The emission of photon without being aided by any external agency is called _____.*

a) light amplification *b) induced absorption*

c) stimulated emission *d) spontaneous emissions.* **VTU : Feb.-09**

(Ans. : d)

Q.2 *If n_1 is the number density of lower energy E_1 and n_2 is the number density of higher energy E_2 then $n_2 > n_1$ is called _____.*

a) thick population *b) inverted population*

c) normal population *d) no population.* **VTU : Feb.-09**

(Ans. : b)

Q.3 *Supply of energy to atoms for excitation is called _____.*

a) glowing *b) bombarding* *c) incidenting* *d) pumping*

VTU : Feb.-09

(Ans. : d)

Q.4 *Important characteristic of laser beam is _____.*

a) interference *b) diffraction* *c) dispersion* *d) coherence*

VTU : Feb.-09, Jan.-12

(Ans. : d)

Q.5 *Pumping process used in diode laser is _____.*

a) optical pumping *b) forward bias* *c) electric discharge* *d) none of these*

VTU : July-09, 13, Jan.-12, 13

(Ans. : b)

Q.6 *The life time of an atom on a metastable state is of the order, _____.*

a) a few seconds *b) unlimited* *c) a nonosecond* *d) few millisecond*

VTU : Aug.-09, June-12, Jan.-13

(Ans. : d)

Q.7 *The purpose of the optical resonator in a laser is _____.*

a) to provide cover to the active medium

b) to provide path for atoms

c) to provide selectivity of photons

d) to send laser in specified direction.

VTU : Aug.-09

(Ans. : d)

Q.8 *The life time of an atom at the ordinary excited state is of the order of _____.*

a) few milliseconds b) few nanosecond c) few microsecond d) unlimited

VTU : Feb.-10, June-10

(Ans. : b)

Q.9 *The relation between Einstein's coefficients A and B is _____.*

a) $\frac{8\pi h\lambda^3}{c^3}$ *b)* $\frac{8\pi h^2\nu^3}{c^3}$ *c)* $\frac{8\pi h\nu^3}{c^3}$ *d)* $\frac{8\pi h\nu^3}{c^2}$

VTU : Feb.-10

(Ans. : c)

Q.10 *The number of modes of standing waves in the resonant cavity of length 1 m, if He-Ne laser operating at wavelength of 6328 Å is _____.*

a) 3.16×10^6 *b)* 1.58×10^6 *c)* 3.16×10^8 *d) none of these*

VTU : Feb.-10

(Ans. : a)

Q.11 *From a broken hologram which is 10 % of the original, if reconstruction of image is being done, then _____.*

a) only 10 % of information of the object can be obtained

b) complete information of the object is obtained

c) no information of the object can be obtained

d) none of these

(Ans. : b)

Q.12 *Population inversion ensures that there will be _____.*

a) stimulated emission *b) light amplification*

c) spontaneous emission *d) coherence*

VTU : June-12

(Ans. : b)

Q.13 *Stimulated emission of radiation in laser is responsible for its _____.*

a) unidirectionality b) high intensity c) high frequency d) coherence

(Ans. : d)

Q.14 *If E_1 is energy of atom in ground state and E_2 in excited state, a photon of energy $E_2 - E_1$ when incident on an atom in energy state E_1 will _____.*

a) be absorbed *b) be emitted*

c) cause stimutated emission *d) none of the above*

(Ans. : a)

Q.15 *If a photon of energy $E_2 - E_1$ is incident on an atom in energy state E_2, having ground state E_1, the photon will _____.*

a) be absorbed *b) be reflected*

c) cause stimulated emission *d) none of the above*

(Ans. : c)

Q.16 *Spontaneous emission of light produces _____.*

a) coherent light *b) incoherent light*

c) unidirectional light *d) none of the above*

(Ans. : b)

Q.17 *In holography, the real image is _____.*

a) at the same place as the object *b) magnified*

c) inverted *d) all the above*

(Ans. : c)

Q.18 *Image is stored on a hologram in the form of _____.*

a) interference pattern *b) diffraction pattern*

c) photograph *d) none of the above*

VTU : Aug.-11, June-12

(Ans. : a)

Q.19 *Holography records _____.*

a) only amplitude *b) only phase*

c) both amplitude and phase *d) neither amplitude nor phase*

(Ans. : c)

Q.20 *Photography records _____.*

a) only amplitude *b) only phase*

c) both amplitude and phase *d) neither amplitude nor phase*

VTU : Jan.-12

(Ans. : a)

Q.21 *The energy of photon in laser is _____.*

a) $E > h\nu$ *b)* $E < h\nu$ *c)* $E = h\nu$ *d) none of the above*

(Ans. : c)

Q.22 *If N_1 is the number of atoms in ground state E_1 and N_2 in excited state E_2, _____.*

a) $\frac{N_1}{N_2} = e^{(E_2 - E_1)/kT}$ *b)* $\frac{N_2}{N_1} = e^{(E_2 - E_1)/kT}$

c) $N_1 N_2 = e^{(E_2 - E_1)/kT}$ *d)* $N_1 N_2 = e^{-(E_2 - E_1)/kT}$

(Ans. : a)

Q.23 *Production of laser requires _____.*

a) population inversion *b) pumping mechanism*

c) resonant cavity *d) all the above*

(Ans. : d)

Q.24 *Rate of induced absorption depends on _____.*

a) number of atoms in lower energy state

b) the energy density

c) number of atoms in higher energy state

d) both A and B

VTU : Feb.-11

(Ans. : d)

Q.25 *In semiconductor laser the material used is _____.*

a) any semiconductor

b) direct band gap semiconductor

c) indirect band gap semiconductor

d) not a semiconductor

VTU : Feb.-11

(Ans. : b)

Q.26 *The required condition to achieve laser action in a system is _____.*

a) state of population inversion *b) existence of metastable state*

c) a resonant cavity *d) all the three*

VTU : Feb.-11

(Ans. : d)

Q.27 *In recording the image on the photographic plate the reference beam and the object beam undergo _____ at the photographic plate.*

a) diffraction *b) reflection* *c) interference* *d) polarization*

VTU : Feb.-11

(Ans. : c)

Q.28 *Wavelength of a Laser beam can be used as a standard of _____.*

a) time *b) temperature* *c) angle* *d) length*

VTU : Aug.-11

(Ans. : d)

Q.29 *Which event is likely to takes place, when a photon of energy equal to the difference in energy between two levels is incident in a system ?*

a) Absorption *b) Emission*

c) Absorption and emission *d) None of these*

VTU : Aug.-11

(Ans. : c)

Q.30 *Emission of a photon by an excited atom due to interaction of external energy is called _____.*

a) spontaneous emission *b) stimulated emission*

c) induced absorption *d) light amplification*

VTU : Jan.-12

(Ans. : b)

Q.31 *Which of the following leads coherent light ?*

a) Induced absorption *b) Spontaneous emission*

c) Stimulated emission *d) None of these*

VTU : Jan.-13

(Ans. : c)

Q.32 *The expression for de-Broglie wavelength for an electron under an accelerating potential V is _____.*

a) $\frac{12.26}{\sqrt{V}}$ *m* *b)* $\frac{12.26}{\sqrt{V}}$ $\mathring{A}$ *c)* $\frac{12.26}{\sqrt{V}}$ *nm* *d)* $\frac{12.26}{\sqrt{V}}$ μm

VTU : June-13

(Ans. : b)

Q.33 *A particle moves with velocity* 3×10^6 ms^{-1}. *The wavelength associated with it is 1 nm. Then group velocity of the particle is _____.*

a) $3 \times 10^8 ms^{-1}$ *b)* 3×10^{10} ms^{-1} *c)* 3×10^6 ms^{-1} *d)* 1.5×10^6 ms^{-1}

VTU : June-13

(Ans. : c)

Q.34 *According to the Compton effect, the wavelength of X-rays scattered at an angle greater than zero, _____.*

a) decreases *b) doesn't change* *c) increases* *d) none of these*

VTU : June-13

(Ans. : c)

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6 Optical Fibres

Chapter at a Glance

- Optic fibres consist of a central core of a transparent material surrounded by a cladding of another transparent material. The refractive index of the core is greater than that of the cladding.
- Light introduced into the core suffers multiple total internal reflections due to which it travels long distances without significant loss of energy.
- The angle of incidence θ_0 of light on the core for which the angle of incidence on the core-cladding interface equals the critical angle is known as **angle of acceptance** and $sin\theta_0$ is known as the **numerical aperture**.
- If the fibre is in air,

$$N.A. = sin\theta_0 = \sqrt{n_1^2 - n_2^2}$$

- The **fractional index change** 'Δ' is defined as the ratio of the difference in refractive indices of core and cladding to the refractive index of the core.

$$\Delta = \frac{n_1 - n_2}{n_1}$$

$$N.A. = n_1\sqrt{2\Delta}$$

- The number of modes that can travel in a fibre is determined from V-number which is given by

$$V = \frac{\pi d}{\lambda} \times (N.A.)$$

- The number of modes $= \dfrac{V^2}{2} = \dfrac{\pi^2 d^2}{2\lambda^2} \times (N.A.)^2$

- Step index fibres fibres have a core of homogeneous transparent material of refractive index n_1, and cladding of another homogeneous transparent material of refractive index n_2 $(< n_1)$. There is an abrupt change in refractive index at the core-cladding interface.

- If the core diameter of the step index fibre is of the order of *50* μ*m* to *200* μ*m*, it can transmit a large number of modes. The cladding thickness is typically *20* to *25* μ*m*. These fibres are known as **multimode step index fibres**.
- Step index fibres which have core diameters in the range of *2* μ*m* to *10* μ*m* are known as **single mode step index fibres**.
- In **graded index fibres**, the refractive index is maximum at the axis of the core, decreases gradually upto the cladding and then remains constant throughout the cladding.
- The power loss of optical signals when they travel through optical fibres is known as attenuation.
- The power loss P_L in decibel (dB) is given by

$$P_L = -10 \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

where P_{out} is the output power and P_{in} is the input power.

- Attenuation in optic fibres is due to absorption, scattering and radiation losses.
- Optic fibres are used in telecommunication.

Important Theory Questions and Answers

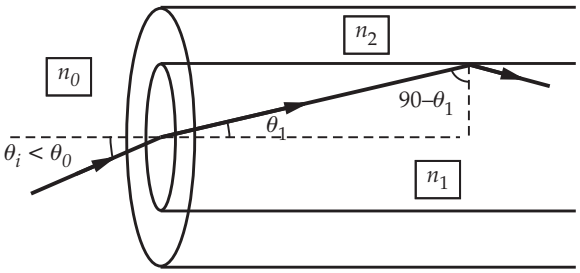
Q.1 *With neat figure derive an expression for N.A. in an optical fibre.*

VTU : Aug.-09, June-10, Marks 6

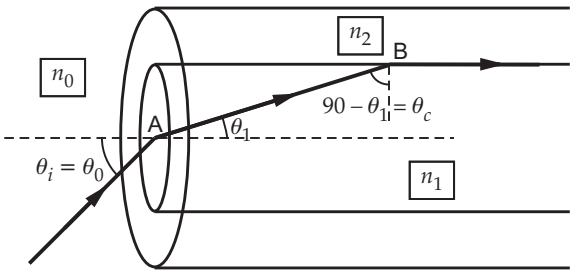
Ans. : If light is transmitted into the core of the optic fibre from a surrounding medium of refractive index n_0 at angle θ_i which is less than the refractive index n_1 of the core, it bends towards the normal. This bending increases the angle of incidence on the core-cladding interface. For very small θ_i, the angle of incidence on the core-cladding interface is more than the critical angle $(90-\theta_1)$ as shown in Fig. 6.1 (a). When θ_i is increased, the angle of incidence on core-cladding interface decreases and becomes equal to the critical angle as shown in Fig. 6.1 (b).

The angle of incidence θ_0 of light on the core for which the angle of incidence on the core-cladding interface equals the critical angle is known as angle of acceptance and $\sin\theta_0$ is known as the numerical aperture.

For $\theta_i > \theta_0$, the angle of incidence on the core-cladding interface is less than the critical angle due to which part of incident light is transmitted as shown in Fig. 6.1 (c). Hence light will be transmitted through the optic fibre for angles of incidence on the core which are less than the acceptance angle. Acceptance angle is the maximum angle of incidence on the core for which total internal reflection takes place inside the core.



(a)



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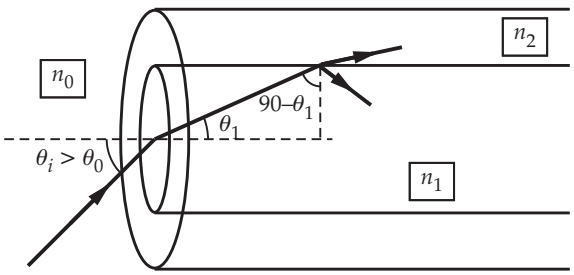

(c)

Fig. 6.1

Using Snell's law at A,

$$n_0 \sin\theta_0 = n_1 \sin\theta_1$$

$$\therefore \quad \sin\theta_0 = \frac{n_1}{n_0} \sin\theta_1 \qquad \text{... (1)}$$

At B, the angle of refraction is 90°.

$$\therefore \quad n_1 \sin(90 - \theta_1) = n_2 \sin 90°$$

$$\therefore \qquad \cos\theta_1 = \frac{n_2}{n_1}$$

As $$\sin\theta_1 = \sqrt{1-\cos^2\theta_1}$$

$$\therefore \qquad \sin\theta_1 = \sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$$

$$\therefore \qquad \sin\theta_1 = \frac{1}{n_1}\sqrt{n_1^2 - n_2^2}$$

Substituting in equation (1),

$$\sin\theta_0 = \frac{n_1}{n_0} \times \frac{1}{n_1}\sqrt{n_1^2 - n_2^2}$$

$$\therefore \qquad \sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \qquad \text{... (2)}$$

As numerical aperture is $\sin\theta_0$,

$$N.A. = \sin\theta_0 = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \qquad \text{... (3)}$$

If the fibre is in air, $n_0 = 1$. Then,

$$N.A. = \sin\theta_0 = \sqrt{n_1^2 - n_2^2} \qquad \text{... (4)}$$

Q.2 *Explain the different types of optical fibre along with the refractive index profile and mode propagation sketches.* **VTU : Feb.-08, Marks 7**

Ans. : Step index fibres

Such fibres have a core of homogeneous transparent material of refractive index n, and cladding of another homogeneous transparent material of refractive index n_2 ($< n_1$). There is an abrupt change in refractive index at the core-cladding interface due to which they are known as step index fibres.

If the core diameter of the step index fibre is of the order of $50\ \mu m$ to $200\ \mu m$, it can transmit a large number of modes as shown in Fig. 6.2. The cladding thickness is typically 20 to $25\ \mu m$. These fibres are known as **multimode step index fibres**. These fibres are cheaper compared to other fibres but not suitable for long distance communication as intermodal dispersion causes significant signal distortion. They are used mainly for short distance communication for which signal distortion due to intermodal dispersion is not significant.

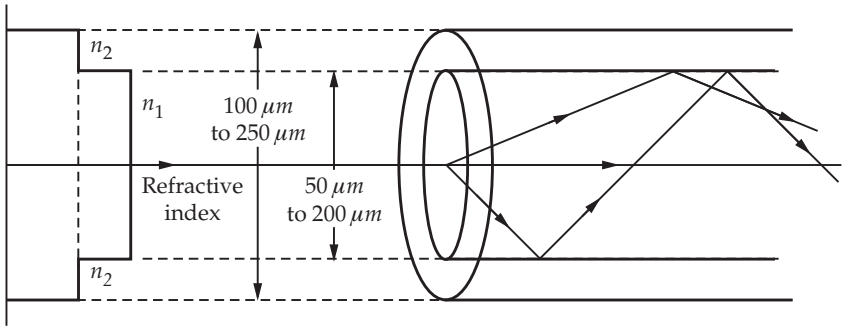

Fig. 6.2

Step index fibres which have core diameters in the range of *2* μ*m* to *10* μ*m* are known as **single mode step index fibres**. These fibres essentially transmit only the axial mode due to very small core diameter as shown in Fig. 6.3. The thickness of the core is typically *25* μ*m* to *30* μ*m*. These fibres are used for long distance communication as intermodal dispersion is eliminated.

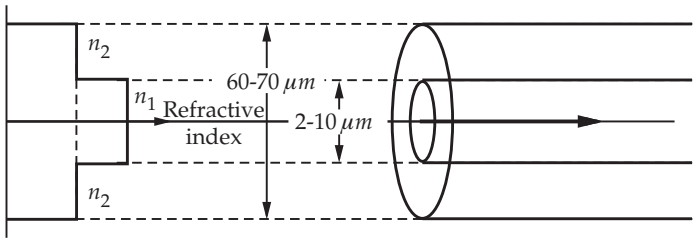

Fig. 6.3

Graded index multimode fibres

In graded index fibres, the refractive index is maximum at the axis of the core, decreases gradually upto the cladding and then remains constant throughout the cladding as shown in Fig. 6.4. The rays travelling at an angle to the axis travel in curved paths due to gradually decreasing refractive index. Although these rays travel longer distances compared to the axial ray, they travel in region of small refractive index and hence travel faster. This reduces the intermodal dispersion. The core diameter is typically about *20* μ*m* to *100* μ*m* with cladding thickness of about *25* μ*m*. They are commonly used for medium distance communication.

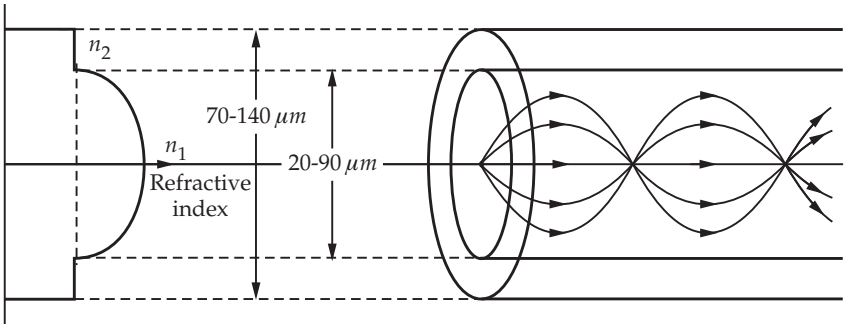

Fig. 6.4

Q.3 *Discuss point to point optical fiber communication system and mention its advantages over the conventional communication systems.* **VTU : June-13, Marks 6**

Ans. : A general block diagram of a communication system based on optic fibres is shown in Fig. 6.5. A non-electrical signal like sound, in telecommunication, is converted into electrical signal by a transducer. The signal is amplified, modulated and the fed to a laser diode which converts the electrical signal into a light signal. Thus the transmitter transmits a light signal. The light signal is transmitted through the optic fibre. In the receiver, the light signal is converted into corresponding electrical signal by a photodetector which is amplified by an amplifier. The amplified electrical signal is then demodulated and in telecommunication is fed to a speaker which converts it into sound signal. In data transfer systems, which transfer digital data from one computer to another, there is no transducer in the transmitter. The input of transmitter and the output of receiver are both electrical signals.

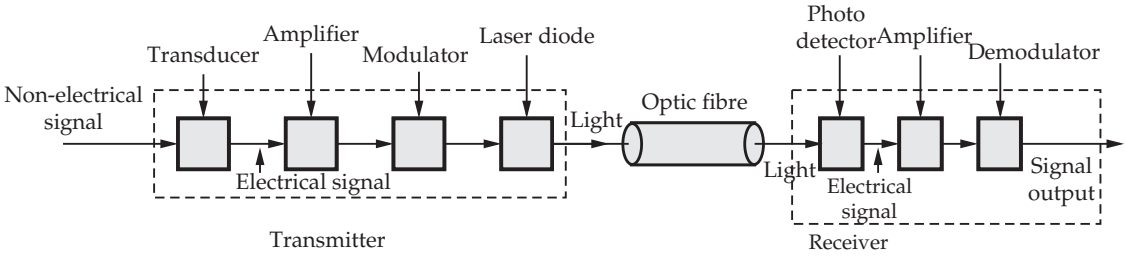

Fig. 6.5

Advantages of optic fibres

1) As the frequency of light is large (~ 10^{14} Hz), large bandwidth is available. Hence more channels become available due to which data transfer speeds are larger.
2) As optic fibres are made up of dielectric materials, the communication system is not affected by electromagnetic disturbances, specially during thunderstorms.
3) There is no cross talk. Signals travelling in one cable do not give rise to induced signals in a nearby cable.
4) Quality of communication is better as the transmission losses are low.
5) As the fibres are very thin and can accommodate more channels, they occupy smaller space compared to copper cables for the same number of communication channels.
6) There is good electrical isolation between transmitter and receiver.
7) They are not affected by corrosion.

Important Solved Examples

Example 6.1 *The angle of acceptance of an optical fibre is 30° when kept in air. Find the angle of acceptance when it is in a medium of refractive index 1.33.*

VTU : Feb.-03, 09, Marks 4, Aug.-11, June-13, Marks 5

Solution : In air,

$$N.A. = \sqrt{n_1^2 - n_2^2} = (\sin\phi_a)_{air}$$

$$\therefore \quad \sqrt{n_1^2 - n_2^2} = \sin 30$$

$$\therefore \quad \sqrt{n_1^2 - n_2^2} = 0.5$$

In medium of refractive index n_0,

$$n_0 \sin\phi_a = \sqrt{n_1^2 - n_2^2}$$

$$\therefore \quad 1.33 \sin\phi_a = 0.5$$

$$\therefore \quad \boxed{\phi_a = 22.08^o}$$

Example 6.2 *Calculate the number of modes an optical fibre will transmit given the following data : $n_{core} = 1.50$, $n_{clad} = 1.48$, core radius = 50 μm, wavelength of light = 1 μm*

VTU : Aug.-07, 08, Marks 4

Solution : The number of modes propagating through the fibre is

$$N = \frac{\pi^2 d^2}{2\lambda^2} \times (N.A.)^2$$

$$d = 2 \times 50 = 100\ \mu m = 100 \times 10^{-6}\ m$$

$$N.A. = \sqrt{n_1^2 - n_2^2} = \sqrt{1.50^2 - 1.48^2}$$

$$\lambda = 1\mu m = 1 \times 10^{-6}\ m$$

$$\therefore \quad N = \frac{1}{2}\left[\frac{\pi \times 100 \times 10^{-6} \times \sqrt{1.50^2 - 1.48^2}}{1 \times 10^{-6}}\right]^2$$

$$\therefore \quad N = 2941.14$$

$$\therefore \quad \boxed{N = 2941}$$

Example 6.3 *An optical fiber of 600 mts long has input power of 120 mW which emerges out with power of 90 mW. Find attenuation in the fiber.* **VTU : Jan.-13, Marks 4**

Solution : The attenuation is given by,

$$P_L = 10\log_{10}\left(\frac{P_{in}}{P_{out}}\right) \text{ in dB}$$

$$P_{in} = 120 \text{ mW}, \quad P_{out} = 90 \text{ mW}$$

$$\therefore \quad P_L = 10\log_{10}\left(\frac{120}{90}\right)$$

$$\therefore \quad \boxed{P_L = 1.25 \text{ dB}}$$

Important Multiple Choice Questions

Q.1 *Loss of power during transmission through optical fibre is called _____.* **VTU : Feb.-09**

a) power loss *b) energy loss* *c) attenuation* *d) modification*

(Ans. : c)

Q.2 *Numerical aperture of the optical fibre depends on _____.*

a) diameter of the fibre *b) acceptance angle*

c) critical angle *d)* η_{core} *material* **VTU : Aug.-09, June-13**

(Ans. : b)

Q.3 *Which of the following is correct ?*

a) Cladding is for providing greater strength

b) Core has higher R.I. than cladding

c) Cladding has higher R.I. than core

d) None **VTU : Aug.-09**

(Ans. : b)

Q.4 *Fractional index change for optical fibre of R.I. of core and cladding 1.563 and 1.498 is _____.*

a) 0.00415 *b) 0.0415* *c) 0.043* *d) 0.004*

VTU : Aug.-09

(Ans. : b)

$$\Delta = \frac{n_1 - n_2}{n_1} = \frac{1.563 - 1.498}{1.563}$$

$$\therefore \Delta = \mathbf{0.0415}$$

Q.5 *In a single mode fibre, the diameter of the core is nearly equal to _____.*

a) 125 μm *b) 100 μm* *c) 50 μm* *d) 10 μm*

VTU : Jan.-10

(Ans. : d)

Q.6 *The numerical aperture of an optical fibre is 0.2 when surrounded by air. The acceptance angle when the fibre is in water of refractive index 1.33 is _____.*

a) 8.21° *b) 8.65°* *c) 0.11°* *d) none of these*

VTU : Jan.-10

(Ans. : b)

For air,

$$N.A. = \sqrt{n_1^2 - n_2^2}$$

$$\therefore \quad \sqrt{n_1^2 - n_2^2} = 0.2$$

In water,

$$n_0 \sin\phi_a = \sqrt{n_1^2 - n_2^2}$$

$$\therefore \quad 1.33 \sin\phi_a = 0.2$$

$$\therefore \quad \phi_a = 8.65^\circ$$

Q.7 *The loss of power by the optical signal through the optical fibre is mainly due to _____.*

a) Reyleigh scattering *b) Raman scattering*

c) Wein's scattering *d) all of these*

VTU : Jan.-10

(Ans. : a)

Q.8 *If light travels from a medium of refractive index 1.5 into air, total internal reflection will take place if the angle of incidence is _____.*

a) 10° *b) 20°* *c) 30°* *d) 50°*

(Ans. : d)

$$\theta_c = \sin^{-1}\left(\frac{1}{1.5}\right) = 41.8^\circ$$

Total internal reflection takes place when angle of incidence is more than 41.8°

Q.9 *In multimode step index fibres, the intermodal dispersion increases if acceptance angle is _____.*

a) increased *b) decreased* *c) 0°* *d) none of the above*

(Ans. : a)

Q.10 *The best fibres for very long distance communication are _____.*

a) multimode step index fibres *b) single mode step index fibres*

c) graded index multimode fibres *d) none of the above*

(Ans. : b)

Q.11 *Attenuation in optic fibres is due to _____.*

a) absorption *b) scattering* *c) radiation loss* *d) all the above*

VTU : June-12

(Ans. : d)

Q.12 *The numerical aperture of an optical fiber in air is 0.32. The numerical aperture in water (n_0 =1/4) is _____.*

a) 0.43 *b) 0.24* *c) 0.64* *d) 0.96*

VTU : Feb.-11
(Ans. : b)

Q.13 *Graded index fiber can be _____.*

a) single mode fiber only *b) multimode fiber only*

c) both single mode and multimode *d) depends on the surrounding medium*

VTU : Feb.-11
(Ans. : a)

Q.14 *If optic fibre is kept in a medium of R.I. μ (>1) instead of air, the acceptance angle _____.*

a) increases *b) decreases* *c) remains constant* *d) none of these*

VTU : Aug.-11, June-12
(Ans. : b)

Q.15 *In graded index fibre, the R.I, of cladding varies _____.*

a) exponentially *b) linearly* *c) parabolically* *d) none of these*

VTU : Aug.-11
(Ans. : c)

Q.16 *The numerical aperture of an optical fibre of which refractive indices of the core and cladding are 1.563 and 1.498, is _____.*

a) 0.446 *b) 1.043* *c) 0.958* *d) none of these*

VTU : Jan.-12
(Ans. : a)

Q.17 *Attenuation is the _____ in power of light as it travels in the fibre.*

a) amplification *b) reduction* *c) gain* *d) none of these*

VTU : Jan.-12
(Ans. : b)

Q.18 *Numerical aperture of an optical fibre depends on _____.*

a) acceptance angle *b) η of cladding* *c) η_{core} of material* *d) All the these*

VTU : June-12
(Ans. : d)

Q.19 *The acceptance angle of optical fiber whose RI of core and cladding of 1.55 and 1.50 respectively is _____.*

a) 32° *b) 45°* *c) 23°* *d) 15°*

VTU : Jan.-13
(Ans. : c)

Q.20 *If red and blue stars emits radiations of continuous wavelengths, then according to Wien's displacement law _____.*

a) blue star is hotter than red star *b) red star is hotter than blue star*

c) both stars are at same temperature *d) difficult to conclude* **VTU : June-13**

(Ans. : a)

Q.21 *The expression for de-Broglie wavelength for an electron under an accelerating potential V is _____.*

a) $\frac{12.26}{\sqrt{V}}$ *m* *b)* $\frac{12.26}{\sqrt{V}}$ $\mathring{A}$ *c)* $\frac{12.26}{\sqrt{V}}$ *nm* *d)* $\frac{12.26}{\sqrt{V}}$ μm

VTU : June-13

(Ans. : b)

Q.22 *A particle moves with velocity* 3×10^6 ms^{-1}. *The wavelength associated with it is 1 nm. Then group velocity of the particle is _____.*

a) $3 \times 10^8 ms^{-1}$ *b)* 3×10^{10} ms^{-1} *c)* 3×10^6 ms^{-1} *d)* 1.5×10^6 ms^{-1}

VTU : June-13

(Ans. : c)

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Notes

7 Crystal Structure

Chapter at a Glance

- The crystalline solids have a long range order i.e. they have a periodic array of atoms.
- A **space or crystal lattice** is defined as an infinite, regular and periodic array of points in three dimensions in which every point has identical surroundings.
- **Basis** is either a single atom or a group of atoms which when attached to every lattice point with identical orientation of identical groups of atoms forms the crystal structure.
- The **unit cell** is smallest geometrical figure in the crystal which when repeated in three dimensions along the three primitive translation vectors forms the crystal structure.
- The **primitive cell** is a unit cell that has only one lattice point per cell.
- The lengths of the three primitive lattice translation vectors in a unit cell and the three interfacial angles (angles between two adjacent faces) in a unit cell are known as the **lattice parameters**.
- There are seven types of crystal systems possible which can have fourteen types of space lattices known as **Bravais lattices**.
- **Miller indices** can be obtained for any plane by using the following procedure :
  1) Determine the intercepts of the plane on the three crystal axes in terms of the lattice parameters a, b and c.
  2) Express the intercepts as multiples of lattice parameters along those axes.
  3) Take reciprocals of these numbers.
  4) Reduce these fractions to a set of smallest integers. This is done by multiplying the three fractions by the L.C.M. of the denominators.
  5) These numbers enclosed in brackets are known as the Miller indices.

- Interplanar spacing in terms of Miller indices :

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

Crystal Structure → Properties ↓	SC	BCC	FCC	Diamond
Number of atoms per unit cell	1	2	4	8
Coordination number	6	8	12	4
Relation between atomic radius and lattice constant	$R = \frac{a}{2}$	$R = \frac{a\sqrt{3}}{4}$	$R = \frac{a}{2\sqrt{2}}$	$R = \frac{a\sqrt{3}}{8}$
Atomic packing factor	0.52	0.68	0.74	0.34

Bragg's law : $2d \sin\theta = n\lambda$

Relation between lattice constant and density of a cubic crystal :

$$a = \left(\frac{n M}{\rho N_A}\right)^{1/3}$$

The structure of barium titanate is known as perovskite structure. The phenomenon in which a solid chemical compound exists in more than one crystalline forms is known as **polymorphism**. When an elemental solid exists in more than one crystalline forms, the phenomenon is known as **allotropy**.

Liquid crystal phase, also known as mesomorphic phase, is an intermediate distinct phase between the solid and the liquid phases of matter. The liquid crystals flow like any other liquid but a certain degree of molecular arrangement is maintained.

Nematic crystals have the property of rotating the plane of vibration of plane polarized light. The amount of rotation is proportional to the thickness of the liquid crystal. The crystal loses this property of rotating the plane of vibration of polarized light if a small potential difference of about 2 - 3 volts is applied.

The liquid crystals are used in liquid crystal displays.

Important Theory Questions and Answers

Q.1 *Derive an expression for interplanar spacing of a crystal in terms of Miller indices.*

VTU : Feb.-07, 09, Aug.-09, Feb.-11, 13, Marks 6

Ans. : Consider a plane ABC making intercepts n_1a, n_2b and n_3c on the three mutually perpendicular axes as shown in Fig. 7.1. Let the adjacent parallel plane be at

the origin. Let the normal ON to these planes defining the distance between the planes make angle θ_x, θ_y and θ_z with *X, Y* and *Z* directions respectively.

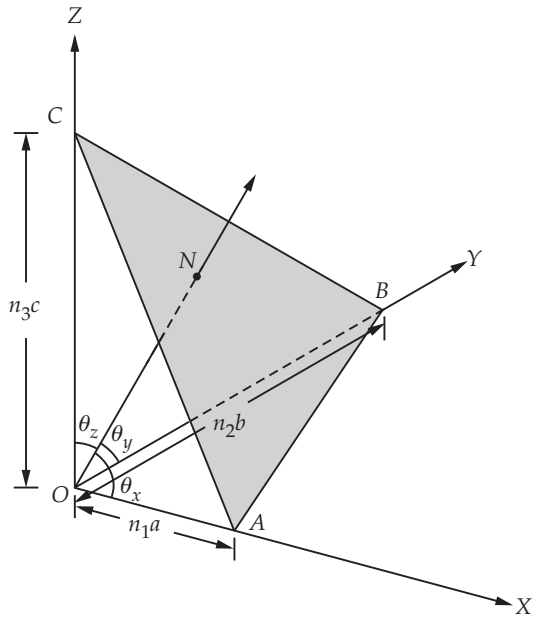

Fig. 7.1

The intercepts of planes : n_1a , n_2b, n_3c

Intercepts in terms of multiples of lattice parameters : n_1 , n_2 , n_3

The reciprocals : $\frac{1}{n_1}, \frac{1}{n_2}, \frac{1}{n_3}$

$$\therefore \qquad h = \frac{1}{n_1}, \quad k = \frac{1}{n_2}, \quad l = \frac{1}{n_3}$$

are the Miller indices of plane ABC.

The direction cosines of the line ON are $\cos\theta_x$, $\cos\theta_y$ and $\cos\theta_z$ given by

$$\cos\theta_x = \frac{ON}{n_1 a} = \frac{d}{n_1 a}$$

$$\therefore \qquad \cos\theta_x = \frac{dh}{a}$$

where $ON = d$ is the interplanar spacing.

$$\cos\theta_y = \frac{ON}{n_2 b} = \frac{d}{n_2 b}$$

$$\therefore \qquad \cos\theta_y = \frac{dk}{b}$$

$$\cos\theta_z = \frac{ON}{n_3 c} = \frac{d}{n_3 c}$$

$$\therefore \quad \cos\theta_z = \frac{dl}{c}$$

The relation between the three direction cosines is

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

$$\therefore \left(\frac{dh}{a}\right)^2 + \left(\frac{dk}{b}\right)^2 + \left(\frac{dl}{c}\right)^2 = 1$$

$$\therefore d^2\left[\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}\right] = 1$$

$$\therefore \quad d = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{b^2} + \frac{l^2}{c^2}}} \quad \text{... (1)}$$

For a cubic crystal,

$$a = b = c$$

$$\therefore \quad d = \frac{1}{\sqrt{\frac{h^2}{a^2} + \frac{k^2}{a^2} + \frac{l^2}{a^2}}}$$

$$\therefore \quad d = \frac{a}{\sqrt{h^2 + k^2 + l^2}} \quad \text{... (2)}$$

Q.2 *Define packing fraction. Calculate packing fraction for sc and bcc structures.*

VTU : Feb.-11, 13, Marks 6

Ans. : The Atomic Packing Factor (APF) is defined as the ratio of the volume occupied by the atoms in the unit cell to the total volume of the unit cell.

The volume occupied by atoms in unit cell

= Number of atoms per unit cell × Volume of an atom

If n = Number of atoms per unit cell,

R = Radius of an atom,

Volume of an atom $= \frac{4}{3}\pi R^3$

$\therefore$ Volume occupied by atoms in a unit cell $= n \times \frac{4}{3}\pi R^3$

If a is the lattice constant for cubic unit cell, the total volume of the unit cell is a^3.

$$\therefore \qquad APF = \frac{n \times \frac{4}{3}\pi R^3}{a^3} \qquad \text{... (1)}$$

For different unit cells, we use the number of atoms per unit cell and the relation between R and a to find the APF.

Simple cubic cell :

For a simple cubic cell, the number of atoms per unit cell is $n = 1$ and the relation between R and a is

$$R = \frac{a}{2}$$

Substituting in equation (1),

$$APF = \frac{1 \times \frac{4}{3}\pi\left(\frac{a}{2}\right)^3}{a^3}$$

$$= \frac{\pi}{6}$$

$$\therefore \qquad APF = 0.52$$

Thus the atomic packing factor of a simple cubic cell is 0.52 which means that 52 % of the volume of the unit cell is occupied by atoms.

Body centred cubic cell :

For a body centred cubic cell, the number of atoms per unit cell is $n = 2$ and the relation between R and a is

$$R = \frac{a\sqrt{3}}{4}$$

Substituting in equation (1),

$$APF = \frac{2 \times \frac{4}{3}\pi\left(\frac{a\sqrt{3}}{4}\right)^3}{a^3}$$

$$= \frac{\pi\sqrt{3}}{8}$$

$$\therefore \qquad APF = 0.68$$

Thus the APF of a body centred cubic cell is 0.68 which means that 68 % of the volume of the unit cell is occupied by atoms.

Q.3 *Describe the construction and working of a twisted nematic mode liquid crystal display.*

Ans. : Nematic crystals have the property of rotating the plane of vibration of plane polarized light. The amount of rotation is proportional to the thickness of the liquid crystal. The crystal loses this property of rotating the plane of vibration of polarized light if a small potential difference of about 2 - 3 volts is applied. In the twisted nematic mode LCD, the thickness of the liquid crystal is such that it rotates the plane of vibration by 90°.

The liquid crystal is placed between two thin glass plates separated by spacers. A thin layer of transparent metallic coating is deposited on the inner side of the glass plates to provide electrical contact. This assembly is placed between two polaroids with mutually perpendicular axes and a mirror is placed beyond the second polaroid as shown in Fig. 7.2.

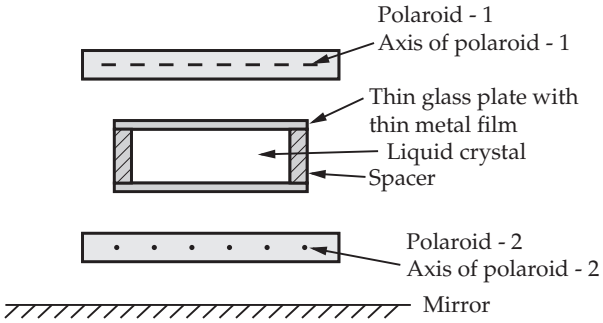

Fig. 7.2

When unpolarized light is incident on polaroid-1, it transmits plane polarized light with vibrations parallel to its axis. When no potential difference is applied across the crystal, it rotates the plane of vibration 90° as shown in Fig. 7.3. Hence the plane of vibration becomes parallel to the axis of polaroid-2 which transmits this light completely. This light is reflected by the mirror and transmitted by polaroid-2. When this light passes through the crystal again, its plane of vibration is again rotated by 90° due to which the plane of vibration becomes parallel to axis of polaroid-1. Hence there is reflected light when no potential difference is applied across the crystal and the display appears bright.

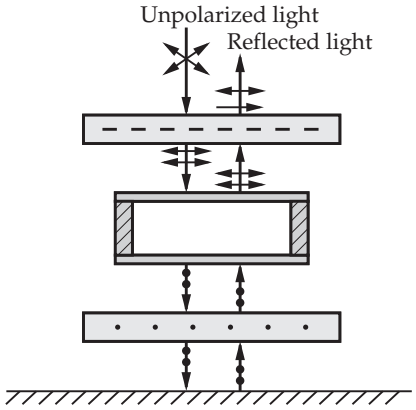

Fig. 7.3

When potential difference is applied across the crystal, it does not rotate the plane of vibration as shown in Fig. 7.4. Hence the plane of vibration remains perpendicular to the axis of polaroid-2 which blocks this light completely. Hence there is no reflected light when potential difference is applied across the crystal and the display appears dark.

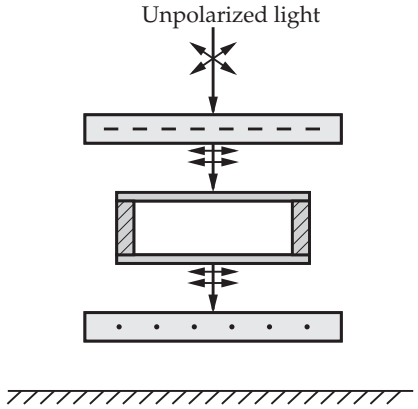

Fig. 7.4

In seven segment displays, each segment has a structure similar to that shown in Fig. 7.2. The seven segments are arranged in the form of an '8' as shown in Fig. 7.5. By applying voltage across a set of segments, different numbers can be displayed. The displayed number appears dark against a bright background. Such displays are used in calculators as their power consumption is very low. The only disadvantage is that they require an external light source.

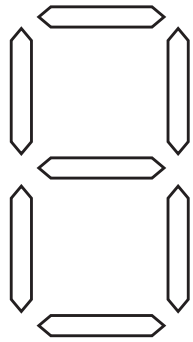

Fig. 7.5

Important Solved Examples

Example 7.1 *Draw the following planes in a cubic unit cell : (1 3 2), (1 $\bar{1}$ 0), (0 1 0), (1 $\bar{2}$ 1).* **VTU : Aug.-06, Marks 8**

Solution : For (*0 1 0*) plane the intercepts are

$$x = \frac{1}{0} = \infty, \quad y = \frac{1}{1} = 1, \quad z = \frac{1}{0} = \infty$$

The crystal plane is shown in Fig. 7.6.

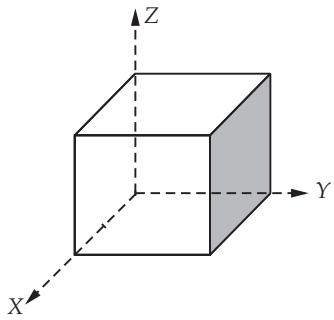

Fig. 7.6

For (*1 3 2*) plane the intercepts are

$$x = 1; \quad y = \frac{1}{3}; \quad z = \frac{1}{2}$$

The crystal plane is shown in Fig. 7.7 (a).

For (*1 $\bar{1}$ 0*) plane, the intercepts are

$$x = 1; \quad y = -1; \quad z = \infty$$

The crystal plane is shown in Fig. 7.7 (b).

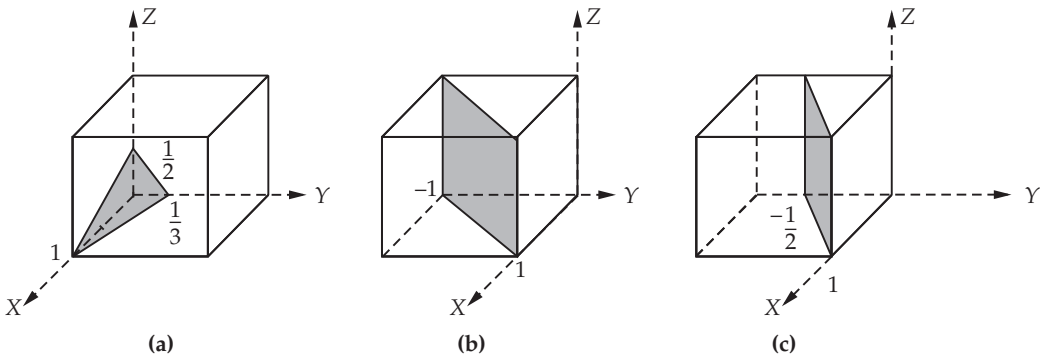

Fig. 7.7

For $(1\ \bar{2}\ 0)$ plane, the intercepts are

$$x = 1; \quad y = -\frac{1}{2}; \quad z = \infty$$

The crystal plane is shown in Fig. 7.7 (c).

Example 7.2 *Monochromatic X-rays of wavelength 0.82 Å undergo first order Bragg reflection from a crystal of cubic lattice with lattice constant 3 Å at a glancing angle of 7.855 Å. Identify the possible planes which give rise to this reflection in terms of their Miller indices.* **VTU : Jan.-12, Marks 6**

Solution : By Bragg's law,

$$2d \sin\theta = n\lambda$$

$$\theta = 7.855^{\circ}, \quad \lambda = 0.82\ \text{Å}, \quad n = 1$$

$$\therefore \quad 2d \sin 7.855 = 1 \times 0.82$$

$$\therefore \quad d = 3\ \text{Å}$$

$$d_{hkl} = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

$$a = 3\ \text{A}^{\circ}$$

$$\therefore \quad 3 = \frac{3}{\sqrt{h^2 + k^2 + l^2}}$$

$$\therefore \quad \sqrt{h^2 + k^2 + l^2} = 1$$

$$\therefore \quad h = 1,\ k = l = 0 \quad \text{or} \quad h = 0,\ k = 1,\ l = 0$$

$$\text{or} \quad h = 0,\ k = 0 \quad \text{and} \quad l = 1.$$

$\therefore$ The possible planes are (1 0 0) , (0 1 0) and (0 0 1)

Important Multiple Choice Questions

Q.1 *The co-ordination number in the case of simple cubic crystal structure is _____.*

VTU : Feb.-09, Aug.-10

a) 12 *b) 6* *c) 2* *d) 1*

(Ans. : b)

Q.2 *Which of the following metal crystallizes in fcc structure ?*

a) Aluminium *b) Zinc* *c) Sodium* *d) Calcium chloride*

VTU : Feb.-09

(Ans. : a)

Q.3 *The Miller indices of the plane parallel to X and Y axes are _____.*

a) (1 0 0) *b) (0 1 0)* *c) (0 0 1)* *d) (1 1 1)*

VTU : Feb.-09, 12, 11

(Ans. : c)

Q.4 *The relation between atomic radius and lattice constant in FCC structure is _____.*

a) $a = 2r$ *b)* $a = 2\sqrt{2}\,r$ *c)* $a = \frac{\sqrt{3}\,r}{4}$ *d)* $\frac{4\,r}{\sqrt{3}}$

VTU : Aug.-09, June-12

(Ans. : b)

Q.5 *A.P.F. of diamond crystal structure is _____.*

a) 0.68 *b) 0.74* *c) 0.52* *d) 0.34*

VTU : Aug.-09

(Ans. : d)

Q.6 *A crystal of tetragonal lattice is _____.*

a) $a = b = c$ *b)* $a \neq b \neq c$ *c)* $a = b \neq c$ *d)* $a \neq b = c$

VTU : Aug.-09, June-12

(Ans. : c)

Q.7 *For every rotation by an angle* θ *in Bragg's spectrometer, detector turns by an angle _____.*

a) θ *b)* 3θ *c)* $\theta / 2$ *d)* 2θ

VTU : Aug.-09, Jan.-12, June-13

(Ans. : d)

Q.8 *Which one of the following crystal is an example of monoclinic ?*

a) SnO_4 *b)* $NaCl$ *c)* $CaSO_4$ *d)* $CuSO_4$

VTU : Feb.-10

(Ans. : c)

Q.9 *In a simple cubic lattice the ratio* $d_{100} : d_{110} : d_{111}$ *is _____.*

a) $6 : 3 : 1$ *b)* $\sqrt{3} : \sqrt{6} : 1$ *c)* $6 : 3 : \sqrt{2}$ *d)* $\sqrt{6} : \sqrt{3} : \sqrt{2}$

VTU : Feb.-10

(Ans. : d)

Q.10 *Which one of the following Bravais lattices is not found in cubic crystal ?*

a) Simple cube *b) Face centred* *c) Body centred* *d) Base centred*

VTU : Feb.-10

(Ans. : d)

Q.11 *The packing fraction of diamond crystal structure is _____.*

a) 34 % *b) 52 %* *c) 68 %* *d) 74 %*

VTU : Feb.-10, June-12

(Ans. : a)

Q.12 *Which of the following unit cells is a primitive cell ?*

a) simple cubic *b) bcc* *c) fcc* *d) none of the above*

VTU : June-12

(Ans. : a)

Q.13 *Miller indices of a plane parallel to X and Z-axes are _____.*

a) (1 0 0) *b) (0 1 0)* *c) (0 0 1)* *d) (1 1 1)*

(Ans. : b)

Q.14 *Miller indices of a plane parallel to Y and Z axes are _____.*

a) (1 0 0) *b) (0 1 0)* *c) (0 0 1)* *d) (1 1 1)*

(Ans. : a)

Q.15 *The number of atoms per unit cell in diamond is _____.*

a) 1 *b) 2* *c) 4* *d) 8* **VTU : Jan.-12**

(Ans. : d)

Q.16 *The number of atoms per unit cell in sc is _____.*

a) 1 *b) 2* *c) 4* *d) 8*

(Ans. : a)

Q.17 *The number of atoms per unit cell in bcc is _____.*

a) 1 *b) 2* *c) 4* *d) 8*

(Ans. : b)

Q.18 *The number of atoms per unit cell in fcc is _____.*

a) 1 *b) 2* *c) 4* *d) 8*

(Ans. : c)

Q.19 *The co-ordination number in body centred cubic cell is _____.*

a) 6 *b) 8* *c) 12* *d) 2*

(Ans. : b)

Q.20 *The co-ordination number in face centred cubic cell is _____.*

a) 2 *b) 6* *c) 8* *d) 12*

(Ans. : d)

Q.21 *Relation between atomic radius 'R' and lattice constant 'a' in simple cubic cell is _____.*

a) $R = \frac{a}{2}$ *b)* $R = \frac{a\sqrt{3}}{4}$ *c)* $R = \frac{a\sqrt{3}}{2}$ *d)* $R = \frac{a}{2\sqrt{2}}$

(Ans. : a)

Q.22 *Relation between atomic radius 'R' and lattice constant 'a' in body centred cubic cell is _____.*

a) $R = \frac{a}{2}$ *b)* $R = \frac{a\sqrt{3}}{4}$ *c)* $R = \frac{a\sqrt{3}}{2}$ *d)* $R = \frac{a}{2\sqrt{2}}$

(Ans. : b)

Q.23 *A.P.F. for a simple cubic cell is _____.*

a) 0.34 *b) 0.52* *c) 0.68* *d) 0.74*

(Ans. : b)

Q.24 *A.P.F. for a body centred cubic cell is _____.*

a) 0.34 *b) 0.52* *c) 0.68* *d) 0.74*

(Ans. : c)

Q.25 *A.P.F. for a face centred cubic cell is _____.*

a) 0.34 *b) 0.52* *c) 0.68* *d) 0.74*

(Ans. : d)

Q.26 *The number of Bravais lattices in cubic crystal system is _____.*

a) 1 *b) 2* *c) 3* *d) 4*

(Ans. : c)

Q.27 *Which of the following has diamond structure ?*

a) Silicon *b) KCl* *c) KBr* *d) NaCl*

(Ans. : a)

Q.28 *The number of Bravais lattice is _____.*

a) 7 *b) 10* *c) 12* *d) 14*

(Ans. : d)

Q.29 *The relation for angles between axes of a triclinic crystal is _____.*

a) $\alpha = \beta = \gamma = 90^o$ *b)* $\alpha \neq \beta \neq \gamma = 90^o$ *c)* $\alpha \neq \beta \neq \gamma \neq 90^o$ *d)* $\alpha = \beta = \gamma \neq 90^o$

VTU : Feb.-11

(Ans. : c)

Q.30 *The coordination number for a face centered cubic lattice is _____.*

a) 12 *b) 8* *c) 6* *d) 26*

VTU : Feb.-11

(Ans. : a)

Q.31 *The packing factor of fcc structure is _____.*

a) 52 % *b) 68 %* *c) 92 %* *d) none of these*

VTU : Feb.-11

(Ans. : d)

Q.32 *Four types of Bravais lattices are obtained in _____.*

a) rhombhohedral system *b) orthorhombic system*

c) triclinic system *d) monoclinic system*

VTU : Aug.-11

(Ans. : b)

Q.33 *In BCC structure, the packing density of crystal is equal to _____.*

a) $\frac{\sqrt{3\pi}}{8}$ *b)* $\frac{\sqrt{3}\,\pi}{8}$ *c)* $\frac{3\pi}{8}$ *d)* $\frac{3\sqrt{3\pi}}{8}$

VTU : Aug.-11

(Ans. : b)

Q.34 *Which of the following has greatest packing fraction _____.*

a) simple cubic *b) body centred cubic*

c) face centred cubic *d) all have equal packing fraction*

VTU : Aug.-11
(Ans. : c)

Q.35 *The space lattice of diamond is _____.*

a) simple cubic *b) body centred cubic*

c) face centred cubic with two atoms/unit cell

d) face centred cubic with four atoms/unit cell

VTU : Aug.-11
(Ans. : d)

Q.36 *The grating space of calcite is 3.036 $\mathring{A}$ and for the first order Bragg reflection, the glancing angle is 12°. The path difference between the rays is _____.*

a) 0.63 $\mathring{A}$ *b) 6.3 $\mathring{A}$* *c) 1.262 $\mathring{A}$* *d) 12.62 $\mathring{A}$*

VTU : Jan.-12
(Ans. : c)

Q.37 *The lattice parameters $a = b \neq c$ and $\alpha = \beta = \gamma = 90°$ represent, _____.*

a) cubic *b) tetragonal* *c) rhombohedral* *d) orthorhombic*

VTU : Jan.-13
(Ans. : b)

Q.38 *Which of the following has least packing fraction ?*

a) SC *b) BCC* *c) FCC* *d) Diamond*

VTU : Jan.-13
(Ans. : d)

Q.39 *In a simple cubic lattice $d_{111} : d_{110} : d_{100}$ = _____.*

a) $\sqrt{6} : \sqrt{3} : \sqrt{2}$ *b) $\sqrt{2} : \sqrt{6} : \sqrt{3}$* *c) $\sqrt{2} : \sqrt{3} : \sqrt{6}$* *d) $\sqrt{3} : \sqrt{6} : \sqrt{2}$*

VTU : Jan.-13
(Ans. : c)

Q.40 *If red and blue stars emits radiations of continuous wavelengths, then according to Wien's displacement law _____.*

a) blue star is hotter than red star *b) red star is hotter than blue star*

c) both stars are at same temperature *d) difficult to conclude*

VTU : June-13
(Ans. : a)

Q.41 *According to the Compton effect, the wavelength of X-rays scattered at an angle greater than zero, _____.*

a) decreases *b) doesn't change* *c) increases* *d) none of these*

VTU : June-13
(Ans. : c)

❑❑❑

8 Shock Waves

Chapter at a Glance

- A **shock wave** is a pressure wave, like the sound wave, but with a very steep rise in pressure.
- It has a compression wave front and gives rise to sudden and large changes in density and temperature across it.
- If the flow velocity of a fluid at a point is less than the speed of sound in that fluid, the flow is said to be **subsonic flow**.
- If the flow velocity of a fluid at a point is greater than the speed of sound in that fluid, the flow is said to be **supersonic flow**.
- Mach number is the ratio of speed of an object through a fluid to the speed of sound in the fluid at that point.

$$M = \frac{V}{a}$$

where V is the speed of object and a is the speed of sound in the fluid. For fluid flow, V is the velocity of fluid at a point.

- **Incompressible flow :** If the Mach number is less than 0.3 **($M < 0.3$)**, there are very small changes in density that can be neglected. Hence the flow can be considered to be incompressible.
- **Subsonic flow :** For Mach number between 0.3 and 0.8 **($0.3 < M < 0.8$)**, the flow is said to be subsonic.
- **Transonic flow :** For Mach number between 0.8 and 1.2 **($0.8 < M < 1.2$)**, the flow is said to be transonic.
- **Supersonic flow :** For Mach number between 1.2 and 5.0 **($1.2 < M < 5.0$)**, the flow is said to be supersonic.
- **Hypersonic flow :** If the Mach number is greater than 5.0, the flow is said to be hypersonic.
- **Stationary shock wave :** In a stationary shock wave, the shock front remains stationary with respect to a fixed observer.

- **Moving shock wave :** Such shock waves travel in fluids which are either stationary or move with small speeds with respect to the observer.
- **Normal shock wave :** If the shock wave is perpendicular to the flow of gas, it is known as normal shock wave.
- **Oblique shock wave :** If the shock wave is at some angle (other than 90°) to the flow of gas, it is known as oblique shock wave.
- By principle of conservation of mass,

$$\rho_1 V_1 = \rho_2 V_2 = \text{Constant}$$

By principle of conservation of momentum,

$$p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2 = \text{Constant}$$

By principle of conservation of energy,

$$u_1 + p_1 v_1 + \frac{V_1^2}{2} = u_2 + p_2 v_2 + \frac{V_2^2}{2} = \text{Constant}$$

or

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} = \text{Constant}$$

The state of a fluid obtained by reducing its velocity to zero isentropically is known as the **stagnation state**.

The properties of a fluid in the stagnation state are known as **stagnation properties**.

Prandtl - Meyer equation :

$$V_1 V_2 = a^{*2}$$

Across a normal shock, the velocity changes from supersonic to subsonic.

If the downstream Mach number is M_2 and the upstream Mach number is M_1,

$$M_2^2 = \frac{\frac{2}{\gamma - 1} + M_1^2}{\frac{2\gamma}{\gamma - 1} M_1^2 - 1}$$

The pressure ratio is

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma + 1} M_1^2 - \frac{\gamma - 1}{\gamma + 1}$$

The temperature ratio is

$$\frac{T_2}{T_1} = \left(\frac{1}{M_1^2} + \frac{\gamma-1}{2}\right)\left(\frac{2\gamma}{\gamma-1}M_1^2 - 1\right)\frac{2(\gamma-1)}{(\gamma+1)^2}$$

Rankine - Hugoniot Relations :

$$\frac{T_2}{T_1} = \frac{\frac{p_2}{p_1}\left[1 + \frac{\gamma-1}{\gamma+1}\frac{p_2}{p_1}\right]}{\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}}$$

and

$$\frac{\rho_2}{\rho_1} = \frac{\frac{\gamma+1}{\gamma-1}\frac{p_2}{p_1} + 1}{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}$$

Shock tube is a device used to study shock waves in the laboratory.

Important Theory Questions and Answers

Q.1 *Derive the Rankine - Hugoniot equations.*

Ans. : The pressure ratio in terms of the upstream Mach number is given by

$$\frac{p_2}{p_1} = \frac{2\gamma}{\gamma+1}M_1^2 - \frac{\gamma-1}{\gamma+1} \quad \ldots$$

$$\therefore \quad M_1^2 = \frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}\right)$$

The temperature ratio is given by

$$\frac{T_2}{T_1} = \left(\frac{1}{M_1^2} + \frac{\gamma-1}{2}\right)\left(\frac{2\gamma}{\gamma-1}M_1^2 - 1\right)\frac{2(\gamma-1)}{(\gamma+1)^2}$$

$$= \left\{\left[\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}\right)\right]^{-1} + \frac{\gamma-1}{2}\right\}\left\{\frac{\gamma+1}{\gamma-1}\left(\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}\right) - 1\right\}\left\{\frac{2(\gamma-1)}{(\gamma+1)^2}\right\}$$

$$= \left\{\left[\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1} + \frac{\gamma-1}{\gamma+1}\right)\right]^{-1} + \frac{\gamma-1}{2}\right\}\left\{\frac{\gamma+1}{\gamma-1}\frac{p_2}{p_1}\right\}\left\{\frac{2(\gamma-1)}{(\gamma+1)^2}\right\}$$

$$= \left\{\left[\frac{\gamma+1}{2\gamma}\left(\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}\right)\right]^{-1}+\frac{\gamma-1}{2}\right\}\left\{\frac{2}{\gamma+1}\frac{p_2}{p_1}\right\}$$

$$= \left\{\left[\frac{\frac{2\gamma}{\gamma+1}}{\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}}\right]+\frac{\gamma-1}{2}\right\}\left\{\frac{2}{\gamma+1}\frac{p_2}{p_1}\right\} = \frac{\frac{4\gamma}{(\gamma+1)^2}\frac{p_2}{p_1}}{\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}}+\frac{\gamma-1}{\gamma+1}\frac{p_2}{p_1}$$

$$= \frac{\frac{4\gamma}{(\gamma+1)^2}\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}\left(\frac{p_2}{p_1}\right)^2+\left(\frac{\gamma-1}{\gamma+1}\right)^2\frac{p_2}{p_1}}{\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}}$$

$$= \frac{\frac{p_2}{p_1}\left[\frac{4\gamma}{(\gamma+1)^2}+\frac{\gamma-1}{\gamma+1}\frac{p_2}{p_1}+\left(\frac{\gamma-1}{\gamma+1}\right)^2\right]}{\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}} = \frac{\frac{p_2}{p_1}\left[\frac{4\gamma+(\gamma-1)^2}{(\gamma+1)^2}+\frac{\gamma-1}{\gamma+1}\frac{p_2}{p_1}\right]}{\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}}$$

$$\therefore \quad \frac{T_2}{T_1} = \frac{\frac{p_2}{p_1}\left[1+\frac{\gamma-1}{\gamma+1}\frac{p_2}{p_1}\right]}{\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}} \qquad ...(1)$$

For perfect gas

$$p = \rho R T$$

$$\therefore \quad \rho = \frac{p}{RT}$$

$$\therefore \quad \frac{\rho_2}{\rho_1} = \frac{p_2}{p_1}\times\frac{T_1}{T_2} = \frac{p_2}{p_1}\times\frac{\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}}{\frac{p_2}{p_1}\left[1+\frac{\gamma-1}{\gamma+1}\frac{p_2}{p_1}\right]} = \frac{\frac{p_2}{p_1}+\frac{\gamma-1}{\gamma+1}}{1+\frac{\gamma-1}{\gamma+1}\frac{p_2}{p_1}}$$

$$\therefore \quad \frac{\rho_2}{\rho_1} = \frac{\frac{\gamma+1}{\gamma-1}\frac{p_2}{p_1}+1}{\frac{\gamma+1}{\gamma-1}+\frac{p_2}{p_1}} \qquad ...(2)$$

Equations (1) and (2) are known as Rankine - Hugoniot equations.

Q.2 *State the applications of shock waves.*

Ans. : The applications of shock waves are as follows :

1) They are used in the treatment of kidney stones.
2) Shock waves are used to treat fractures as they activate the healing process in tendons and bones.
3) Shock waves are used in metallurgy for compaction of ceramic powders.
4) The shock waves are used in wave disc engines to transfer energy from a fluid having large energy to a fluid having small energy.
5) Shock waves develop when objects like jets and rockets move at supersonic speeds. Hence the shock waves are studied to develope designs for jets, rockets and high speed turbines.
6) Shock waves develop in pipes through which fluids flow at supersonic speeds. Hence they are studied to develop designs for ducts and pipes.

Important Multiple Choice Questions

Q.1 *Shock wave is a _____ wave.*

a) electromegnetic *b) gravity* *c) pressure* *d) sound*

(Ans. : c)

Q.2 *The thickness of a shock wave is of the order of _____.*

a) 1 m *b) 100 m* *c)* 10^{-10} *m* *d)* 10^{-7} *m*

(Ans. : d)

Q.3 *Waves of frequency more than 20 kHz are known as _____ waves.*

a) ultrasonic *b) supersonic* *c) subsonic* *d) sonic*

(Ans. : a)

Q.4 *Waves travelling with a speed more than the local speed of sound are known as _____ waves.*

a) ultrasonic *b) supersonic* *c) subsonic* *d) sonic*

(Ans. : b)

Q.5 *Waves travelling with a speed less than the local speed of sound are known as _____ waves.*

a) ultrasonic *b) supersonic* *c) subsonic* *d) sonic*

(Ans. : c)

Q.6 *Waves travelling with a speed equal to the local speed of sound are known as _____ waves.*

a) ultrasonic *b) supersonic* *c) subsonic* *d) sonic*

(Ans. : d)

Q.7 *If V is the speed of an object and the speed of sound is a, the Mach number is _____.*

a) $V \times a$ *b)* $\frac{a}{V}$ *c)* $\frac{V}{a}$ *d)* $\frac{V^2}{a^2}$

(Ans. : c)

Q.8 *For waves travelling with a speed equal to the local speed of sound the Mach number is _____.*

a) 1 *b)* < 1 *c)* > 1 *d) 0*

(Ans. : a)

Q.9 *For waves travelling with a speed exceeding the local speed of sound the Mach number is _____.*

a) 1 *b)* < 1 *c)* > 1 *d) 0*

(Ans. : c)

Q.10 *For waves travelling with a speed less than the local speed of sound the Mach number is _____.*

a) 1 *b)* < 1 *c)* > 1 *d) 0*

(Ans. : b)

Q.11 *The equation* $\rho_1 V_1 = \rho_2 V_2 =$ *constant represents _____ .*

a) conservation of mass *b) conservation of momentum*

c) conservation of energy *d) none of the above*

(Ans. : a)

Q.12 *The equation* $p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2 =$ *constant represents _____ .*

a) conservation of mass *b) conservation of momentum*

c) conservation of energy *d) none of the above*

(Ans. : b)

Q.13 *The equation* $h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} =$ *constant represents _____ .*

a) conservation of mass *b) conservation of momentum*

c) conservation of energy *d) none of the above*

(Ans. : c)

Q.14 *The downstream Mach number in a shock wave is _____ the upstream Mach number.*

a) less than *b) equal to*

c) greater than *d) none of the above*

(Ans. : a)

Q.15 *There is smooth variation of pressure in _____.*

a) shock wave *b) sound wave* *c) both a) and b)* *d) neither a) nor b)*

(Ans. : b)

Q.16 *There is steep increase in pressure in _____.*

a) shock wave *b) sound wave* *c) both a) and b)* *d) neither a) nor b)*

(Ans. : a)

Q.17 *There is variation of pressure in _____.*

a) shock wave *b) sound wave* *c) both a) and b)* *d) neither a) nor b)*

(Ans. : c)

Q.18 *Rankine - Hugoniot relations give relation between _____.*

a) temperature ratio and pressure ratio *b) density ratio and pressure ratio*

c) temperature ratio and density ratio *d) both a) and b)*

(Ans. : d)

Q.19 *Shock waves are produced in a fluid when upstream Mach number is _____ .*

a) equal to 1 *b) less than 1*

c) more than 1 *d) zero*

(Ans. : c)

Q.20 *Rankine - Hugoniot relations are derived using _____ .*

a) conservation of mass *b) conservation of momentum*

c) conservation of energy *d) all the above*

(Ans. : d)

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Notes

9 Nanotechnology

Chapter at a Glance

- Nanoparticles can be defined as an aggregate of atoms between *1 nm* and *100 nm* with dimensions less than the characteristic length of some physical phenomena.
- Density of states is the possible electron quantum energy states between energies E and $E+dE$ per unit volume.
- The density of states in a bulk material is proportional to $E^{1/2}$ and hence increases with energy.
- For a quantum well, which is a material reduced in one dimension to nano - scale, the density of states for a given quantum state is constant.
- For a quantum wire, which is a material reduced in two dimensions to nano - scale, the density of states for a given quantum state decreases with increasing energy.
- For a quantum dot, only some discrete energy states are allowed for the electrons.
- The nanoparticles are synthesized either by using the 'top-down' approach or the 'bottom-up' approach.
- Nanoparticles can be synthesized using ball mill method and sol-gel method.
- Fullerene also known as Buckminster Fullerene, is a molecule of carbon having 60 atoms.
- The structure of carbon nanotube (CNT) can be imagined as a cylinder formed by rolling a graphene sheet and then closing it on both sides by fullerene hemispheres.
- Carbon nanotubes can be fabricated using arch method and chemical vapour deposition.
- Carbon nanotubes are good conductors of heat and electricity. They are highly elastic and have high tensile strength.
- The resolving power of electron microscopes is 10^5 times larger than optical microscopes.
- The electron microscopes makes use of magnetic lenses to focus electrons.

- The transmission electron microscope is used to study viruses, bacteria, atomic and crystal structures and colloidal particles.
- SEM gives high resolution images of surfaces as it uses electrons to produce images.

Important Theory Questions and Answers

Q.1 *What are nano materials ? Write a note on carbon nanotubes.*

VTU : Jan.-08, July-09, Marks 7

Ans. : Nanoparticles can be defined as an aggregate of atoms between *1 nm and 100 nm* with dimensions less than the characteristic length of some physical phenomena. Nanomaterials consist of nanoparticles.

In graphite, each carbon atom is linked to three other carbon atoms through covalent bonds forming hexagonal rings connected to each other to form plane sheets known as graphene. These plane sheets are linked to adjacent layers through the weak Van der Waal's forces. The structure of carbon nanotube (CNT) can be imagined as a cylinder formed by rolling a graphene sheet and then closing it on both sides by fullerene hemispheres.

The CNTs are of two types :

1) Single walled nanotubes (SWNT) which can be imagined as a single atom thick graphite sheet, i.e. graphene, rolled into a cylinder and capped by fullerene hemispheres.
2) Multiwalled nanotubes (MWNT) which can be considered as nanotubes within nanotubes.

Structure

The graphene sheet can be rolled in different ways to get the three types of SWNTs : Zig-zag, armchair and chiral.

Properties of carbon nanotubes

1) **Mechanical :** The carbon nanotubes are highly elastic. The Young's modulus is a measure of the elasticity. The Young's modulus for carbon nanotube is about 1800 GPa whereas it is about 210 GPa for steel.

   Carbon nanotubes exhibit large strength in tension. They are about twenty times stronger than steel. The yield stress is a measure of the strength. High strength steel has yield stress of about 2 GPa whereas it is about 45 GPa for carbon nanotubes. This is however offset by the fact that they are very weak in compression, shear, torsion and bending.

Carbon nanotubes can withstand larger strains than steel. They can be bent without breaking.

2) **Electrical :** The electrical properties of carbon nanotubes depends upon their diameter and chirality. They show electrical properties ranging from semiconductors to those of good conductors. The energy gap decreases as the diameter of the CNT is increased.

   Due to very low resistivity, the heat dissipation in the CNT is very small and hence they can carry much larger currents than the metals.

   The conductivity of a CNT is maximum along its axis and very low in a perpendicular direction. hence they are equivalent to one dimensional conductors.

   At low temperatures, the resistance decreases with increasing magnetic field applied across the CNT. This effect is known as magnetoresistance.

3) **Thermal :** Carbon nanotubes are very good conductors of heat. Their thermal conductivity is more than twice that of diamond. The thermal conductivity also varies with direction. The conductivity is very good along the axis of the tube and very low in a perpendicular direction.

4) **Physical :** CNTs have very high strength to weight ratio. They have low density.

5) **Chemical :** CNTs are chemically more inert compared to other forms of carbon.

Applications

1) Carbon nanotubes can store lithium due to which they can be used in batteries. CNTs can also store hydrogen and hence find potential applications in fuel cells.
2) They are used in the tips for atomic force microscope probes.
3) They are being used to develop flat panel displays for television and computer monitors.
4) They are being used to develop light weight shield for electromagnetic radiation.
5) Field effect transistors are being developed using semiconducting CNTs which can be used to build faster processors for computers. It is estimated that these processors will be 10^4 times faster than the present processors.
6) CNTs are being used to produce light weight materials with higher strength than steel. These can be used in automobile, aircraft and rocket parts.
7) They are used in chemical sensors to detect gases.

Q.2 *Describe the principle, construction and working of a scanning electron microscope.. Also write its applications.*

Ans. : Principle : The surface of a sample is scanned using a high energy beam of electrons. This gives rise to secondary electrons, back-scattered electrons and *X*-rays. The

secondary electrons give topographical information of the surface. Back-scattered electron give topographical information and chemical composition. *X*-rays give the elemental composition.

Construction : An electron gun is used to produce high energy electrons. Two magnetic lenses are used as condenser lenses to convert the diverging electron beam into a fine beam of spot diameter of the order of a few nanometers. A scanning coil is used to deflect the electron beam to scan the sample. The objective lens is used to focus the scanning beam on a desired spot on the sample. The intensities of secondary electron, back-scattered electrons and the *X*-rays are recorded using detectors. The images are then displayed on TV monitor.

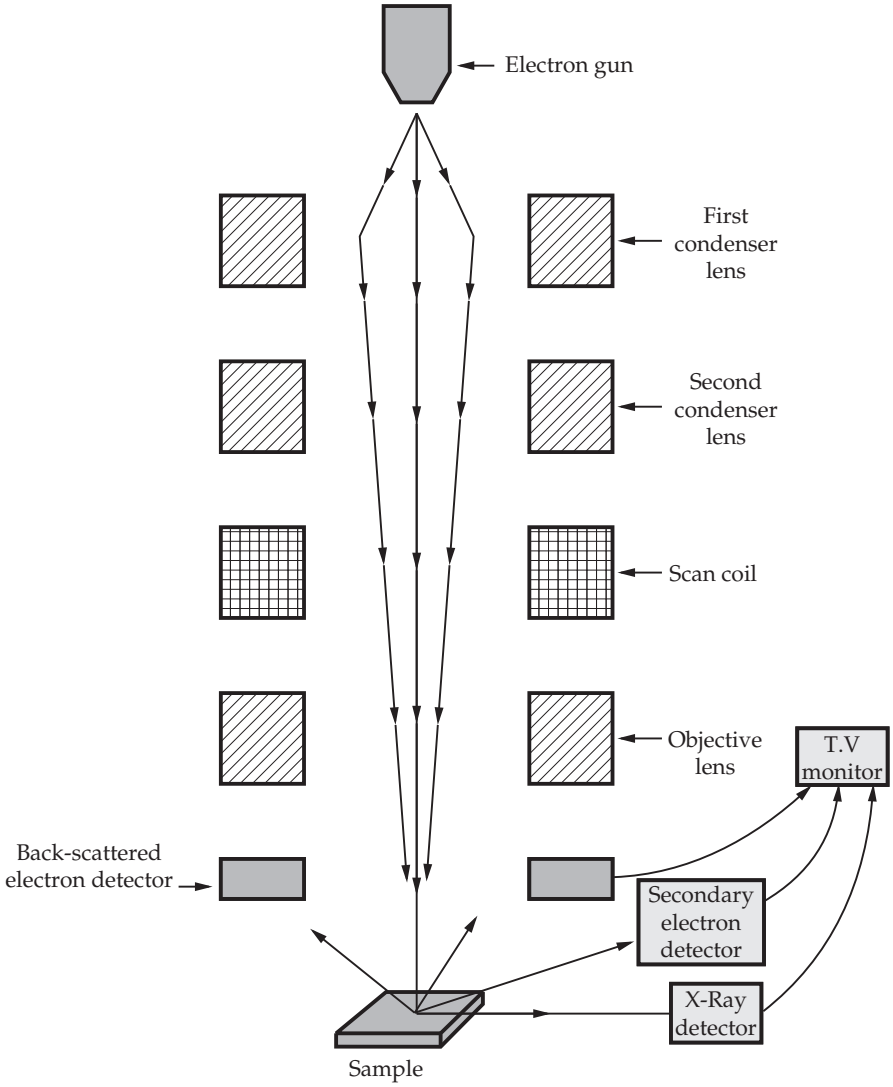

Fig. 9.1

Working : When the high energy electron beam strikes the sample, some electrons are scattered due to elastic scattering (the back-scattered electrons), some electrons are knocked off from the surface (the secondary electrons) and some electrons penetrate deep into the inner shells of the sample atoms to knock off inner shell electrons due to which characteristic *X*-rays are produced. These are detected using detectors and the signals are amplified and displayed on a TV monitor.

Samples are required to be conducting. Non-conducting samples are coated with a thin conducting material.

Applications :

i) SEM gives information about the surface features of the sample with resolution of the order of a few nanometers. This information can be used to study properties like reflectivity and roughness.

ii) SEM images give information about the elements and compounds in the sample and their relative abundance. This is used to study properties like hardness and melting point.

iii) SEM is used to study biological specimens like pollen grains.

iv) SEM is used to study the corroded layers on metal surfaces.

Important Multiple Choice Questions

Q.1 *The state of matter around the nano-size is known as _____ .*

a) solid state *b) liquid state* *c) plasma state* *d) mesoscopic state*

VTU : Feb.-09, June-10, Aug.-11

(Ans. : d)

Q.2 *If the reduction is in two directions, the resultant structure will be in one dimension which is called _____ .*

a) reduced structure *b) thin wire* *c) quantum wire* *d) enlarged structure*

VTU : Feb.-09, June-10

(Ans. : c)

Q.3 *Carbon nanotubes are molecular structures of _____ .*

a) graphite sheet *b) graphene sheet* *c) plastic* *d) none*

VTU : Aug.-09

(Ans. : b)

Q.4 *As per the scaling laws, the frequency of operation increases with _____ .*

a) decrease of size *b) increase of size* *c) constant* *d) none*

VTU : Aug.-09

(Ans. : a)

Q.5 *Which one of these does not represent a type of carbon nanotube ?*

a) Arm chair *b) Wavy* *c) Zig-zag* *d) Arch discharge*

VTU : Feb.-10

(Ans. : d)

Q.6 *The bulk material reduced in two directions is known as _____ .*

a) quantum dot *b) quantum wire* *c) film* *d) reduced structure*

(Ans. : b)

Q.7 *Fullerene is a _____ .*

a) molecule *b) atom* *c) chemical compound* *d) none of the above*

(Ans. : a)

Q.8 *Which of the following statement is false regarding a CNT ?*

a) They have high mechanical strength *b) They have high resistivity*

c) They can store hydrogen *d) None of the above*

(Ans. : b)

Q.9 *The nanostructure reduced in only one direction is known as _____ .*

a) quantum dot *b) quantum wire* *c) quantum well* *d) film*

(Ans. : c)

Q.10 *The nanostructure reduced in all three dimensions is known as _____ .*

a) quantum dot *b) quantum wire* *c) quantum well* *d) film*

(Ans. : a)

Q.11 *In a carbon nano tube, the bond between the carbon atoms is _____ .*

a) metallic *b) ionic* *c) hydrogen* *d) covalent*

VTU : Feb.-11

(Ans. : d)

Q.12 *Fullerene is _____ .*

a) a sheet of carbon atoms rolled up into long tube

b) sixty carbon atoms arranged in the shape of a football

c) one dimensional array of atoms

d) three dimensional array of atoms

VTU : Feb.-11

(Ans. : b)

Q.13 *The typical size of nano material is between*

a) 1 - 10 nm *b) 10 - 50 nm* *c) 1 - 100 nm* *d) 1 - 1000 nm*

VTU : Feb.-11

(Ans. : c)

Q.14 *At nanoscale, the surface to volume ratio is _____ .*

a) very low *b) very high* *c) equal to one* *d) equal to five*

(Ans. : b)

Q.15 *The density of states in a bulk material is proportional to _____.*

a) E *b)* $E^{1/2}$ *c)* $E^{-1/2}$ *d)* E^{0}

(Ans. : b)

Q.16 *The density of states in a quantum well is proportional to _____.*

a) E *b)* $E^{1/2}$ *c)* $E^{-1/2}$ *d)* E^{0}

(Ans. : d)

Q.17 *The density of states in a quantum wire is proportional to _____.*

a) E *b)* $E^{1/2}$ *c)* $E^{-1/2}$ *d)* E^{0}

(Ans. : c)

Q.18 *Confinement of electron along any direction leads to _____ of energy.*

a) increase *b) decrease* *c) quantization* *d) none of the above*

(Ans. : c)

Q.19 *Which of the following is used to produce images in an electron microscope ?*

a) Light *b) Electrons* *c) X - rays* *d) Gamma rays*

(Ans. : b)

Q.20 *_____ lenses are used in electron microscopes.*

a) Magnetic *b) Electrostatic* *c) Optical* *d) Electromagnetic*

(Ans. : a)

Q.21 *The method of producing nanoparticles by breaking down a material in bulk form is known as _____ .*

a) top - down approach *b) bottom - up approach*

c) either a) or b) *d) none of the above*

(Ans. : a)

Q.22 *The method of producing nanoparticles by bringing individual atoms and molecules together is known as _____ .*

a) top - down approach *b) bottom - up approach*

c) either a) or b) *d) none of the above*

(Ans. : b)

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Notes